MATHEMATICAL MODELING OF COOKIE AND CRACKER OVENS

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Abstract

In my paper "The Mathematical Modeling ofCookie and Cracker Ovens", I have presented in detail three such mathematical models. The present document insists mostly on two models, namely Engineer Model and Engineer Research Model as being the ones based mostly on a scientific approach.
***Keywords**: mathematical model, cookie, crakers*

Introduction

The models are applied to three categories of cookies, clasifid as such based on their content of sugar and fat: **Semisweet cookies** (very crisp texture and breaks cleanly with a distinct snap), **Short dough** (breaks more easily than the semisweet types, and in doing so sheds more fragments) and **Cream Cracker** (blistered and friable)

Table1: The following table presentes the recipes for thess three types of cookies:

Ingredients	Semisweet	Short dough	Cream Cracker
Flour	100	100	100
Water	19.6	11.9	32.1
Sugar	20.8	29.5	
Fat	16.1	32.1	12.5
Glucose	1.3	-	-
Yeast	-	-	1.8
Salt	1.2	1.1	1.4
Sodium Bicarbonate	0.5	0.4	0.2
Ammonia Bicarbonate	1.1	0.2	-
SMP (is skimmed milk powder)	-	1.8	-
SMS (sodium metabisulfite)	0.02	-	-
Cream Powder	0.2	-	-

A model is a representation of the real thing. A ***mathematical model*** of a real system is a set of equations that emulates the behavoir of the system in some respect. The real system will have a number of inputs and outpus that are time – varying physical or chemical parameters. The model will have inputs and outputs that are numerical analogues of the real ones, although the model may not possess all the inputs and outputs of the real system. There are three main mathematical models:

- The Engineer Model
- The Bake Model
- The Research Model

The Engineer Models of the oven are used mainly for three reasons:

- The first is as an aid for tendering, in which a set of mathematical rules is used to work out technical details of an oven that will meet the client's specification.
- The second occasion for using a model is at the design stage;
- The third engineer's model is for research.

The Bake Model

Such a model would see the whole bakery as a set of inputs under the direct control of the bakery staff and a set of outputs that are essentially properties of the dough pieces and finished cookies, including the packets. For the oven in particular the inputs would include temperatures, turbulence, and extraction damper settings and band speed.

The Research Model

The weakness of the models just discussed is their largely empirical nature. Empiricism calls for substantial amounts of on – site measurement that is expensive and often unreliable; furthermore it may have to be repeated every time a new factor is encountered, for example, a new product or a different oven. A scientific knowledge of the mechanisms that determine the important parameters of the product would enable all or, at any rate, some of the on – site measurement to be bypassed.

The main feature of this type of model as opposed to the others is that it looks into the mass and the course of moisture out of it. At the same time it should explain the change in volume of cookie, the change in texture, change in color, and other property of concern to the bakery as a whole.

Experimental

The ovens are built up in sections partly as a convenience to the engineers, since a number a short sections is obviously easier to transport and erect than one single monolithic structure (figure 1). However, the existence of independently controllable sections does offer the baker the opportunity of facility that is much valued. There are three types of ovens:

- **Type 1:** The multiburner oven in which gas is actually burned in the baking chamber, above and below the product. (figure 2)
- **Type 2**: The forced convection oven in which gas is burned in a separate chamber, and the hot combustion products then blown from nozzles at the product. (figure 3)
- **Type 3**: The indirect oven in which the combustion chamber is completely sealed off from the baking chamber.(figure 4)

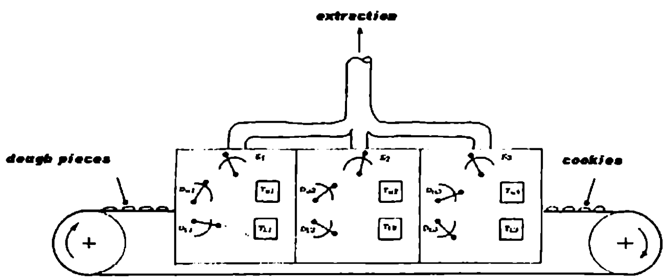

Fig. 1: Line drawing of a three – section cookie oven in side elevation, showing notional control points.(key : E_n - extraction controller, *n*th section; D_{un} - damper controlling turbulence, upper *n*th section; T_{Ln} - temperature controller, lower *n*th section.)

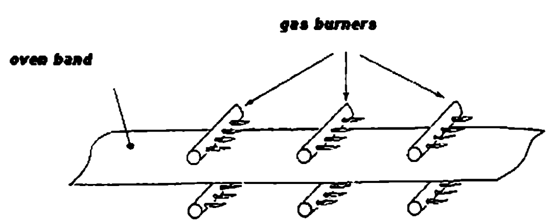

Fig. 2: Line drawing showing greatly simplified heating principle of a multiburner oven

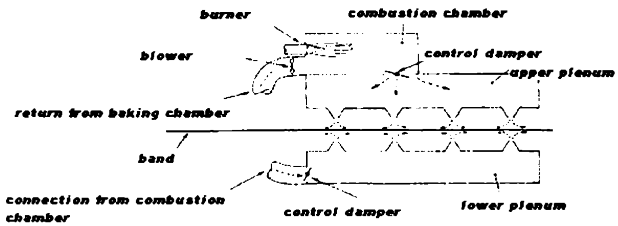

Fig. 3: Line drawing showing greatly simplified interior of a forced convection oven

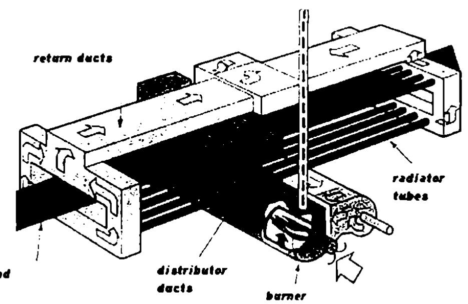

Fig.4: Line drawing showing simplified interior of an indirectly heated oven

Instrumentation of the ovens:

- Temperature
- Turbulence
- Humidity
- Pressure
- Baking Time
- Cookie Temperatures
- Oven Heat - Flux

Dough Piece Properties

- Dough Piece Thickness
- Ingredient Proportions
- Weigth

Cookie Properties : weigth, thicness, length, width, top crust - color, bottom – crust color, extent of hollow bases and moister content; other proprietes that the person who constructs the model should have in mind are: the texture, the spontaneous cracking, the surface, the humidity distribution within the cracker and flavors.

Result and Discussions

1. The engineer's model

The plan area of the oven is calculated from the client's required number of cookies per hour, from prior knowledge of the time of bake, and from the area of band taken up by the cookie, including the spaces between.Thus, if the number of cookies per hour is N_c, the baking time is t_c, and the area taken up by a cookie is A_c then the total working area of the oven is $N_c * t_c * A_c / 60$.

2. The engineer's research model

The starting point for such a model might well be the diagram of figure 5, which shows the lateral cross – section of an oven in which the cookies on the band (b-b´) are assumed to travel into the plane of the paper. Heat is supplied to the oven trom heat sources (S1-S1´) and (S2-S2´), which are slim rods or pipes mounted above and below the band; they may be gas burners that provide lateral ribbons of flame; they may be sources of hot gas blown through slots in a plenum chamber (in which case the fans F_1 and F_2 would not be necessary); or they may be pipes, sealed from the baking chamber, in which hot flue gases are flowing.

Conditions in the oven are assumed to be in the steady state, that is, all temperatures, air speeds, and pressures are constant. It follows that heat flows into and out of section are in balance.

From the point of mass balance, it is simplest to consider the type of oven for which flue gases are sealed from the baking chamber. We also assume that forced turbulence is evenly distributed along the oven. A simple heat – balance equation is:

$$q_{cs} = \lambda dm_c / dt + m_c c_p dT_{mc} / dt \qquad (11\text{-}1)$$

where :

q_{cs} – the total heat enteringa unit area of cookie surface in a second;
m_c - the mass of the cookie in unit area;

λ – the latent heat of vaporization of water;
c_p – the mean specific heat of cookie material;
T_{mc} – the mean cookie temperature.

The equation simply says that heat entering the cookie from the oven is used to change water into steam and to raise the temperature of the cookie (this is not entirely accurate as will be seen later); it is important for the engineering model in that it defines the thermal load on the oven. The heat reaching the product is of three sorts, radiant, convective and conductive.

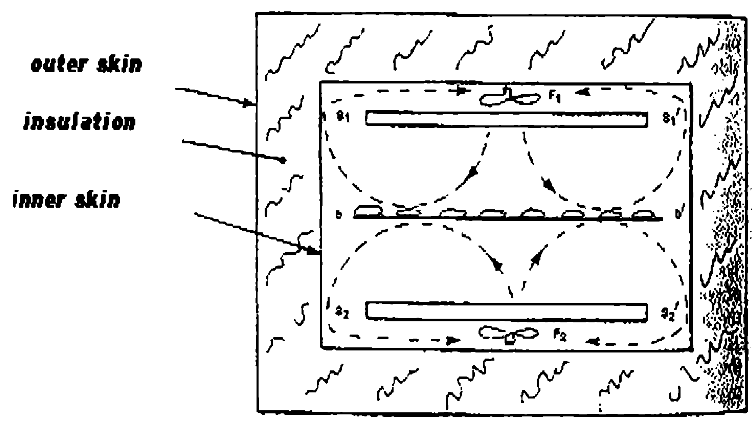

Fig. 5. Simplified cross section of an oven illustrating the engineer's model

Conclusions

An attempt to summarize the engineer's model in a single diagram is given in figure 6. All the surfaces in the oven are interlinked by two apparently independent sets of heat – flux – radiant and convected – but in fact the two sets are mutually dependent through the temperatures of the surfaces, particulary the steel surfaces of the oven structure.

This is a complex network in which a change at any one part will be reflected throughout the whole.

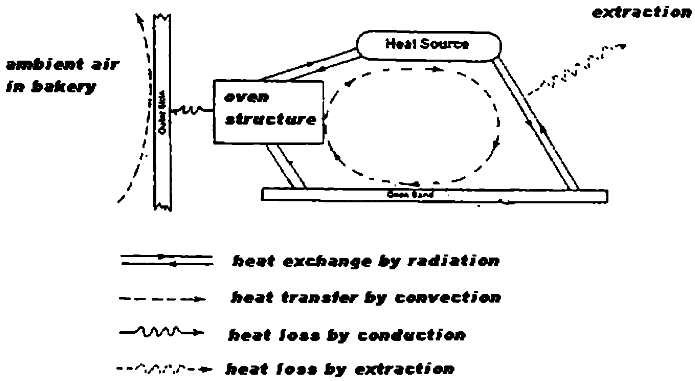

Fig. 6. Diagram illustrating relations between convective and radiative heat fluxes in cookie oven.

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