ON THE GEOMETRY IN THE CROP CIRCLES

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Rezumat:
Cercurile din lanuri reprezintă un fenomen geometric incredibil. În aceste formațiuni din lanuri pot fi găsite spirale, triunghiuri, pentagoane şi pentagrame, hexagoane, etc., fiecare având propria conotație simbolică. Dimensiunile acestor forme furnizează rapoarte simple, cum ar fi 1, 3/2, 5/4, 9/8, etc (numere pe care muzicologii le numesc intervalle perfecte ale gamei do major). În cercurile din lanuri putem găsi câteva teoreme care fac legătura între aceste cercuri şi notele muzicale.

Abstract:
An amazing geometrically phenomenon is the "*crop circles*". Spirals, triangles, pentagons and pentagrams, hexagons etc. are all found in crop formations, each with its own symbolic connotations. The measurements of these patterns provide simple ratios, such as 1, 3/2, 5/4, 9/8, etc (the numbers which musicologists call the 'perfect' intervals of the C major scale). We can find some crop circles' theorems which make a link between crop circles and musical notes.

Résumé:
Les cercles des chaînes représentent un phénomène géométrique in crédible. Ces formations des champs sont des spirales, triangles, pentagones et pentagrammes, hexagones, etc., chacun d'entre eux ayant sa propre connotation symbolique. Les dimensions de ces formes donnent des rapports simples, tels 1, 3/2, 5/4, 9/8, etc. (des numéros que les musicologues appellent intervalles parfaits de la gamme do majeur.) On peut trouver quelques théorèmes dans les cercles des champs qui font la liaison entre ces cercles et les notes musicales.

Introduction

The crop circles have primarily been seen in agricultural regions, and also in the Sahara sands and in Arctic snow. About 10,000 crop circles have been reported worldwide since the 1970s. Most often, they are found in the northern European countries, especially each summer in southwest England in fields of grain and other crops. Many people believe that crop formations carry a symbolically encoded message. An equilateral triangle, for example, can represent completion. The five-pointed star or pentagram (which, like the pentagon, displays the golden mean ratio) symbolizes the human being, since a human figure with outstretched limbs fits within it. The six-pointed star or interlaced triangles represents universal spirit and matter ([24], [19]).

Preliminaries

In the early 1990s, the astronomer Gerald Hawkins ([3]) discovered that the ratios between the areas or diameters of the various elements making up crop patterns clustered around certain whole numbers: 2, 4, 5, 7, 9, 11 and 12. These are the numbers used to generate the diatonic ratios that define the relative frequency of the white notes of a piano keyboard. Hawkins concluded that crop designs demonstrated the remarkable mathematical ability of their creators. He noticed that some of the most visually striking of the crop-circle patterns embodied geometric theorems that express specific numerical relationships among the areas of various circles, triangles, and other shapes ([11], [11], [15], [22]). Hawkins discovered that the various ratios could be expressed in the form of five mathematical theorems based on the principles of Euclidean geometry ([2], [25]). In these theorems concentric circles that touch the sides of a triangle are noticed and as the triangle changes its shape, it generates the special crop-circle patterns. The Gerald Hawkins's theorems are the following:

- **Tangent theorem** ([7]): *If three equal circles are tangent to a common line and their centers can be connected by an equilateral triangle and a circle is circumscribed about the triangle, the ratio of the diameters is 4 to 3.*

- **Triangle theorem** ([7]) *For an equilateral triangle, the ratio of the areas of the circumscribed (outer) and inscribed (inner) circles is 4:1. The area of the ring between the circles is 3 times the area of the inscribed circle.*

- **Square theorem** ([7]) *For a square, the ratio of the areas of the circumscribed and inscribed circles is 2:1. If a second square is inscribed within the inscribed circle of the first, and so on to the mth square, then the ratio of the areas of the original circumscribed circle and the innermost circle is* $2^m:1$.

- **Hexagon theorem** ([7]) *For a regular hexagon, the ratio of the areas of the outer circle and the inscribed circle is 4:3.*

The fifth theorem (general theorem) involves concentric circles which touch the sides of a triangle, and as the triangle changes its shape, it generates the special crop-circle geometric ratios ([7]). In his article ([2]), Dee Gragg presents another new crop circle theorems and then takes the diatonic ratios of all the theorems and relates them to the frequencies of the musical scale. One of these theorems is the following one: "*If three equal circles are tangent to a common line and their centers can be connected by an equilateral triangle and a circle is inscribed within the triangle, the ratio of the diameters is 2:3*".

Euclidian geometry is useful for proving the concentric circular relationships of polygons (equilateral triangle, the square, the pentagon, the hexagon, etc). Crop geometry can be proved by Euclidean logic and constructions and leads generally to a pair or more of diatonic ratios. These ratios are fundamental to music. The existence of these ratios made Hawkins begin looking for geometric relationships among the circles, rings, and lines of several particularly distinctive patterns that had been recorded in the fields. The symbiosis of form and sounds (frequency) dates back to the 4th century BC, when Pythagoras described the harmony of the spheres, which originated from observations of the planets in the heavens ([15], [16]).

Vibration is the source of all sound. The number of vibrations per second is the "frequency. In music, an octave is made up of eight notes. It goes C (do), D (re), E (mi), F (fa), G (sol), A (la), B (si), C (do), and continues up and down on a standard musical instrument (the middle A has a frequency of 440 Hz). The ancient Greeks discovered that to a note with a given frequency only those other notes whose frequencies were integer multiples of the first could be properly combined. If, for example, a note of the frequency 220 Hz was given, the notes of frequencies 440 Hz, 660 Hz, 880 Hz, 1100 Hz and so on sounded best when played together with the first ([4], [5]).

To find the frequency of a note from a higher octave, the frequency is doubled and to find the frequency of a note from a lower octave, the frequency is halved. Frequency can also go in fifths by using the formula $y = 3x/2$, where x is the original note and y is the fifth note above the origina ([15], [16]). 1. Using these ratios between the notes, we find that each note has a unique frequency:

Table no. 1

Musical note	Ratio	Frequency (Hz)
C (do)	1	264
D (re)	9/8	297
E (mi)	5/4	330
F (fa)	4/3	352
G (sol)	3/2	396
A (la)	5/3	440
B (si)	15/8	495
C' (do)	2	528

As Hawkins' Euclidean theorems also produce diatonic ratios, there is a link between the crop circles and musical notes, the latter ones being the by-product of the harmonic laws of sound frequency. In 11 of crop circles structures ([22]), Hawkins found ratios of small whole numbers that precisely matched the ratios defining the diatonic scale. These ratios produce the eight notes of an octave in the musical scale corresponding to the white keys on a piano.

New theorems in the crop circles

I find two new theorems in crop circles where diatonic ratios and golden ratio appear.
The first of them was inspired from a design of nested crescents appeared on Morgan's Hill, Wiltshire In August 3-rd 2003 ([23], figure 1), with the inner and outer crescents divided by a number of radial lines.

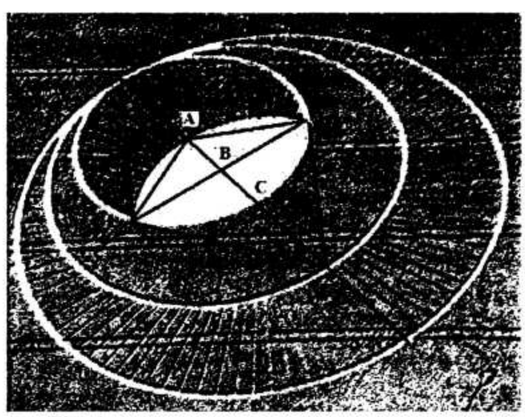

Fig. 1

Theorem 1: If we have three circles which are tangent within at a point M so that the center B of the middle circle is the middle point of the segment determined by the centre A of the smaller circle and the centre C of the greatest circle, then we remark that the diatonic ratios appear in the ratios determined by the segments which have one of the extreme points in the center of one of these circles and the other one on the extreme of diameter of one of this circles, as follows:

$$(1)\frac{MA}{AC}=\frac{MC}{CN}=\frac{MA}{DN}=1,\quad (2)\frac{BN}{CN}=\frac{5}{4},\quad (3)\frac{MB}{MA}=\frac{3}{2},\quad (4)\frac{NB}{MA}=\frac{5}{3},\quad (5)\frac{NC}{AM}=2$$

where MN is the diameter of the greatest circle.

Proof:

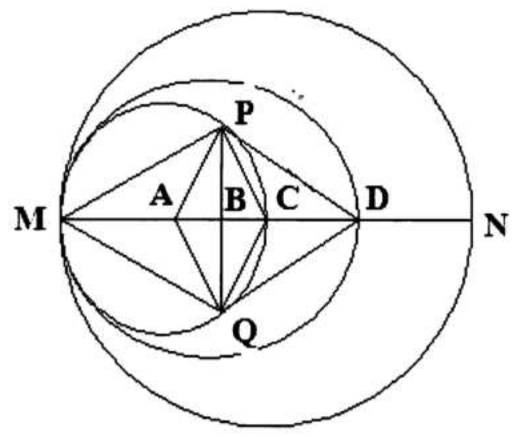

Fig. 2

We remark easily that MA=AC =r (they are radii in the smaller circle), DN=4r-3r=r and MC=CN =2r (are radii in the greatest circle) and from this we have (1).
From BC=AB=r/2, then we have that BN=5r/2. From CN=2r, we obtain (2).
Since A is the intersection of medians in the equilateral triangle inscribe in the smaller circle, then we have (3).
From NB=5r/2 and MA=3r/2, we obtain (4).
From NC=2r and AM=r, it results (5).

Remark: *From the table no. 1, we remark that the ratios 1, 5/4, 3/2, 2 are corresponding to the musical notes C-E-G-C' (respectively), which form the first fundamental accord in the C Major scale (To form the first chord, we start on C, then miss out one note so we get E and then miss out one note again so we get G and C'). The ratio 5/3 is corresponding to the musical notes A.*
Pentagrams and pentagons intrigue many people which think the shape of the pentagram has a special quality. Pentagonal geometry did also appear in crop circles. In July 2003 an elaborate design appeared ([12], figure 3) in the crop field in Avebury Wiltshire.

Theorem 2: *The intersection points of the five circles provide a regular pentagon ABCDE and the intersection points of the diagonal lines provide a regular pentagon MNPQT (see fig. no.3) so that the ratio between the length L of pentagon ABCDE and the length l of pentagon MNPQT is equal with the square of golden number.*

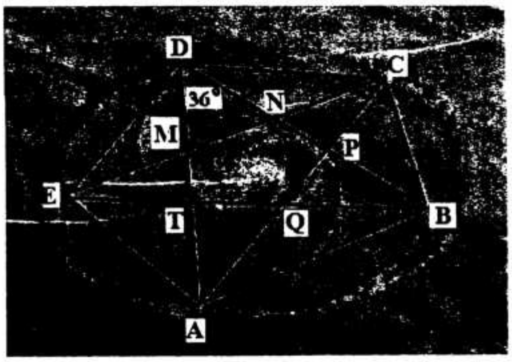

Fig. 3

Proof:

The golden ratio appear in the proportion $\frac{DA}{AB}=\frac{DP}{PB}$ (AP is the bisector of the angle DAB). From $\Delta DAB \sim \Delta APB$ we obtain that $\frac{DA}{AP}=\frac{AB}{PB}$. Thus from DA=DB and DP=AP=AB we have

$$\frac{DB}{DP}=\frac{DP}{PB} \Leftrightarrow DP^2 = PB(DP+PB) \Leftrightarrow \left(\frac{DP}{PB}\right)^2 - \frac{DP}{PB} - 1 = 0 \Leftrightarrow \frac{DP}{PB}=\phi$$

From this, we obtain that $\frac{DA}{AB}=\phi$ and $\cos(D\hat{A}B)=\cos(72^\circ)=\frac{DA}{AB/2}=\frac{\phi}{2}$.

From

$$\frac{DP}{PB}=\phi \Leftrightarrow \frac{DP}{DP-PB}=\frac{\phi}{\phi-1} \Leftrightarrow \frac{AB}{NP}=\frac{\phi}{\phi-1} \Leftrightarrow \frac{NP}{AB}=1-\frac{1}{\phi}$$

and from $\phi^2-\phi=1 \Leftrightarrow 1-\frac{1}{\phi}=\frac{1}{\phi^2}$ we obtain that $\frac{l}{L}=\frac{1}{\phi^2}$, where L is the length of pentagon ABCDE and l is the length of pentagon MNPQT.

Conclusions

In the incredible world of unexplained phenomena, the crop circles are among the first ones. Many people believe that crop formations carry a symbolically encoded message. As the expression of number in space, geometry is inextricably linked to music since the laws of the former govern the mathematical intervals that make up the notes in the western music scale. The geometry and forms that we find in crop circles are strongly related with the music and the diatonic ratios.

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