Journal homepage: www.fia.usv.ro/fiajournal
Journal of Faculty of Food Engineering,
Ştefan cel Mare University of Suceava, Romania
Volume XV , Issue 4 - 2016, pag. 284 - 298

HEAT TRANSFER IN DOWN FLOWING TURBULENT EVAPORATING LIQUID FILMS WITH DEVELOPED WAVY STRUCTURE AND CO-CURRENT STEAM FLOW

Valentyn PETRENKO[1], *Yaroslav ZASYADKO[1]

[1]*National University of Food Technologies, Thermal Fluids and Industrial Refrigeration Dept. 68, Volodymyrska St.; 0160 Kyiv; Ukraine*
Petrenkovp@ukr.net, *yaroslav@nuft.edu.ua
*Corresponding author
Received 2nd December 2016, accepted 29th December 2016

Abstract: *The result of heat transfer modeling in down flowing turbulent films with the developed wavy structure at the regime of evaporation from film interface at free falling and with co-current steam flow, a heat transfer model which takes into account a cyclic process of temperature field relaxation with the periodic mixing of film by big waves, has been presented. The temperature profiles along with the integral heat transfer coefficients (HTC) were obtained as a result of approximate solutions of heat transfer differential equations of in turbulent flows, adapted thus to film flux with large surface waves, utilizing turbulent transport model by M.D.Millionschykov. The results of experimental research of heat transfer processes to water and sugar solutions with concentration up to 70% in the regime of evaporation from the inter-phase are presented.*

Key words: *films, waves, heat transfer, temperature, turbulent, evaporation.*

1. Introduction

The majority of existing heat transfer models applied to the flowing over the vertical surface films at wide variation of regime conditions, parameters and physical properties usually are based upon the application of the flow films' effective film conduction. These variations include: films heated to the boiling point and those that are being heated; turbulent and laminar once and many similar. The list of correlations for heat transfer coefficients (HTC) calculations in films at modes of heating and evaporation from free surface are given in works [1,2,3,4]. Fundamental analysis of heat transfer and hydrodynamic processes in films was made by V.M. Sobin [5]. Analysis is based on integral methods of solving corresponding differential equations for stabilized laminar and turbulent flows as well as those, which develop their structure with boundary conditions of the 1,2,3 and 4 kind. Nonetheless, the given analysis does not take into account the wave structure of film's surface and contains no relevant characteristics of wave film flow.

Nomenclature

y –normal to the heat surface coordinate;
x –longitudinal to the heat surface coordinate;
t –temperature;

$\eta = \frac{y}{\delta}$ –dimensionless transverse coordinates;
$\xi = \frac{x}{\delta}$ –dimensionless longitudinal coordinates;
$\theta(\eta,\xi) = \frac{t(\eta,\xi) - t_{sat}}{t_w - t_{sat}}$ –dimensionless temperature;
δ – film thickness;
Γ_v – volumetric liquid flux;
a –temperature conductivity;
a_t –turbulent temperature conductivity;
ν –cinematic viscosity coefficient;
ν_t –cinematic turbulent coefficient;
t_{sat} –saturation temperature;
t_w – wall temperature;
λ –heat conduction of liquid;
q – heat flow;
C –heating capacity;
ρ –liquid density;
ρ_2 –steam density;
σ – surface tension;
a_{wav} – wave thermal diffusivity coefficient;
f_{bw} –the frequency of the big wave's;
A_{bw} –the amplitude of the big wave's;
c_{bw} –the face velocity of the big wave's;
δ_{ll} –the thickness of the remaining film layer between big waves;
u_i – liquid velocity on film interphase;
$\bar{u}$ – average liquid velocity on film;
$u^* = \sqrt{\frac{\tau_i + \rho g \delta}{\rho}}$ – dynamic velocity;
u_2 – steam velocities;
τ_i –shear stress on the film interface;
g –acceleration of gravity;
$Pe = \frac{4\Gamma_v}{a} = \frac{4\bar{u}\delta}{a}$ –the Peclet number;
$Re = \frac{4\Gamma_v}{\nu}$ –the Reynolds number;
Pr – the Prandtl number;
Pr_t –turbulent Prandtl number;
$We_2 = \frac{\rho_2 u_2^2}{\sqrt{\sigma g \rho_2}}$ –the Weber number;
ς –interface friction coefficient.

The influence of waves on heat transfer in stabilized film was taken into account directly in the articles of E.M.Vorontsov [6,7,8] by introduction of wave thermal diffusivity coefficient a_{wav}

$$q = -\rho C(a + a_{wav})\frac{dt}{dy}, \quad (1)$$

With the use of average characteristic of big waves, such as speed of its transverse pulsations u_b

$$a_{wav} = u_b \delta_{ll} = 2A_{bw} f_{bw} \delta_{ll}, \quad (2)$$

According to the conclusions [9], equations (1) and (2) show a good correlation with the experimental data only in terms at low flow rates and undeveloped turbulence at $Re \le 800$. Later the author [9] concluded that a process of heat transfer through the turbulent film is advisable to present as:

$$q = -\rho C(a + a_{wav} + a_t)\frac{dt}{dy}, \quad (3)$$

The actual character of the film flow with the developed big wave structure proves that the process has a two-dimensional character within the residual films layer and its cyclicity is being determine by the hydrodynamics of big wave movement. According to [10], in case of free falling films over the vertical surface the saturation of wavy movement with fully developed big wave's structure occurs at a distance approximately 2…2.5 m from the film forming devise. An important factor, which seriously effects a hydrodynamic film structure is the fact that big wave's contain a central swirl [11, 12]. Due to this, the bulk of the liquid film is periodically mixed, which in turn causes the velocity, temperature and concentration profiles deformation. Taking into account all mentioned above, the heat transfer in liquid films may be treated as a cyclic process of temperature field relaxation, which happens after each consequent passage of a big wave. This big wave due to the existence of a powerful central swirl, which transports slightly, superheated liquid from the boundary region to the external of film. According to this mechanism the peak of a big wave will be constantly fed with the slightly superheated liquid and transfer it to the wave surface, from which evaporation takes place, thus transferring heat to the steam core. The process of relaxation is stretched along the film movement until a consecutive mixture of liquid happens due to the passage of the consecutive wave. The extended analytical results of heat transfer in liquid down flowing saturated films of solutions based upon the proposed model of cyclic bulk mixing by large waves with the following temperature profile relaxation between the consecutive waves passage are presented in [13].

2.Materials and methods

A direct experimentation of heat transfer in down flowing liquid films heated to the saturation temperatures with sugar solutions as model liquids has been carried out at the experimental unit with the independent formation of phases' mass flow rates and heat flux. The main core of the experimental unit was represented by a stainless still pipe with the inside diameter of 20 mm and 1.8 m long. The experimental tube was separated into the initial 1.5 m stabilization section and 0.3 m measurement section. The down flowing of water (sugar solutions) film has been formed by means of overflowing over the tube's upper rim. In the event of steam-liquid flow modeling, dry saturated steam has been supplied in co-current regime. The liquid falling film has been heated by dry saturated steam which was supplied into outside heating sections attached to the experimental tube. The heating chambers were designed in such a way as to provide an individual heating of the stabilization section and the experimental one. The said sections were hooked up to the individual vacuum-condensation sections which allowed for the keeping of different pressures in each chamber. Such arrangement allowed also maintaining vacuum down to 0.8 bars and thus, vary the temperature head between the heating steam temperature and evaporation temperature. Special probes for taking samples of liquid to determine its concentration and measurements of temperatures were positioned directly after the measurement section. A detailed description of the experimental unit is given in [4], or [13].

3. Results and Discussion.

The bigger is liquid flow rate in the film and the lower its viscosity the turbulence in the film appears and develops. The character of turbulence distribution within the film's cross section and especially in films with the developed wavy structure and massive swirl in big waves is undetermined. In this case it can be assumed that right before the passage of a big wave, the velocity profile will be developed (corresponding to the 1/7 low). The temperature profile will be close to the linear one. Right after the wave passage, similarly to the laminar movement, as a result of mixing the velocity profile will be constant across the film and the temperature profile will become curved, figure 1

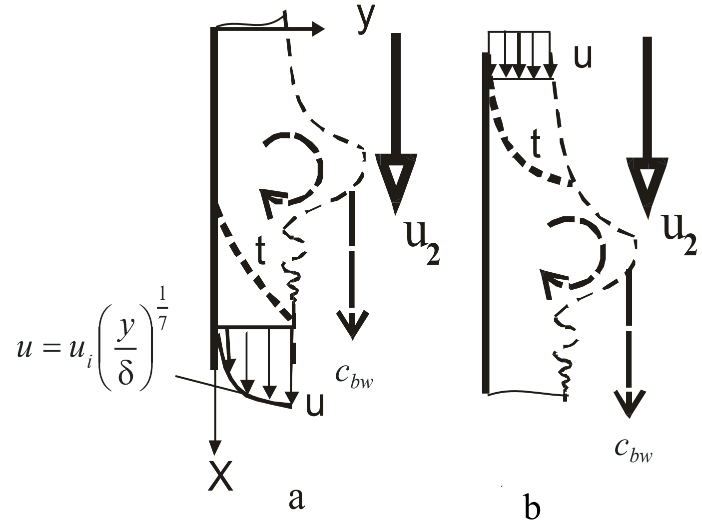

Fig.1 Schematics of a big wave passage on the surface of the turbulent film: velocity and temperature profile (a) - before the big wave passage; (b) – after.

Under these conditions, taking into account, that turbulence intensifies heat transfer, when analysing temperature film fluctuation as a result of film cyclic mixing by big waves the regime $\frac{\partial\theta}{\partial\eta}_{\eta=1} \geq 0$ can be separated for every ξ between consequent big wave passage.

Then the equation of heat transfer in film during the period between big waves passage may be given as:

$$u_i\left(\frac{y}{\delta}\right)^{1/7}\frac{\partial t}{\partial x} = \frac{\partial}{\partial y}(a + a_t)\frac{\partial t}{\partial y} \quad , \tag{4}$$

Taking into account, that, $u = u_i\left(\frac{y}{\delta}\right)^{\frac{1}{7}} = \frac{8}{7}\bar{u}\left(\frac{y}{\delta}\right)^{\frac{1}{7}} = \frac{8}{7}\bar{u}\,\eta^{\frac{1}{7}}$, in the dimensionless form (4) can be written as

$$\frac{8\delta}{7}\bar{u}(\eta)^{1/7}\frac{\partial\theta(\xi,\eta)}{\partial\xi} = \frac{\partial}{\partial\eta}(a + a_t)\frac{\partial\theta(\xi,\eta)}{\partial\eta} \quad , \tag{5}$$

Replacing the left side by average value:

$$\int_0^1 \frac{8\delta}{7}\bar{u}(\eta)^{1/7}\frac{\partial\theta(\xi,\eta)}{\partial\xi}\partial\eta = \delta\bar{u}\frac{\partial\theta_{av}(\xi)}{\partial\xi},$$

Yields: $$\frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}=\frac{\partial}{\partial\eta}\left(1+\frac{a_t}{a}\right)\frac{\partial\theta(\xi,\eta)}{\partial\eta}\quad,\tag{6}$$

Limiting temperature curve, at which the absence of evaporation from film surface takes place, may be found from (6) at the boundary conditions $\eta=0$, $\theta=1$; $\eta=1$, $\frac{\partial\theta}{\partial\eta}=0$. (7)

Whereas, $\frac{a_t}{a}=\frac{\nu_t}{\nu}\frac{Pr}{Pr_t}$, the expression (6) considering (7) take the form

$$\theta(\eta,\xi)=\frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}\left[\int\frac{\eta\, d\eta}{\left(1+\frac{\nu_t}{\nu}\frac{Pr}{Pr_t}\right)}-\int\frac{d\eta}{\left(1+\frac{\nu_t}{\nu}\frac{Pr}{Pr_t}\right)}\right]+c_1,\tag{8}$$

where c_1 – integration constant.

A characteristic feature of the turbulence development in the films is supression of turbulence intensity in the layer close to the wall surface that is proved experimentally by Jepsen [14], Lamurell-Sandal [15], and Ganchev [16]. For free downflowing viscous films the turbulence viscosity profile, generally, is set layer by layer [17]; in the boundary layer– by Van-Drist correlation [18], in the middle layer – by Reyhardt [19] , on the external film surface - Lamurell-Sandal [15]. Another form of turbulence in the film is given by M.D. Millionschikov [20], according to which the laminar layer with a thickness of $\delta_l^+=7.8$ exists, on whose limits $\frac{\nu_t}{\nu}=0$, than a turbulent layer has place with a parabolic profile of turbulent viscosity in the range $\frac{7.8}{\delta^+}\le\eta\le 1$

$$\frac{\nu_t}{\nu}=0.39\left(\eta\delta^+-\delta_l^+\right)(1-\eta),\tag{9}$$

where $\eta_l=\frac{\delta_l}{\delta}=\frac{\delta_l^+}{\delta^+}$; $\delta_l=\frac{7.8\,\nu}{u^*}$; $\delta^+=\frac{\delta u^*}{\nu}$.

Taking into account, that the experimental data of turbulent intensity were obtained in the experiments conducted determining is completed on the inclined surfaces and at big wave's absence [14,15,16], the received results do not represent the real picture of turbulence development in films on the vertical surfaces with developed wave structure and are not in adequate. The only the fact of turbulence suppression near the hard solid walls and interphase surface may be accepted, since a big wave contains powerful central whirl, it's rolling on the film surface should be accompanied with the turbulent pulsation and micro-whirls penetration into the boundary layer. Under these conditions, instead of layer by layer model with boundary laminar layer and turbulent core with parabolic distribution of turbulent intensity (9), [20], the simplified model of equivalent turbulence may be used. Here, the turbulence develops directly from the wall, and its maximum in the central part is determined by comparison of the calculated heat transfer intensity with experimental data.

Under these assumptions the function of turbulence in the film should be given as:

$$\frac{\nu_t}{\nu} = 4\varepsilon_m\left(\eta - \eta^2\right), \qquad (10)$$

where ε_m – turbulence function in the center of film, which should be determined.

If $Pr_t = 1$, the equation (8) should be given as

$$\theta(\eta,\xi) = \frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}\left[\int\frac{\eta\, d\eta}{\left(1+4\varepsilon_m\left(\eta-\eta^2\right)Pr\right)} - \int\frac{d\eta}{\left(1+4\varepsilon_m\left(\eta-\eta^2\right)Pr\right)}\right] + c_1, \quad (11)$$

and its integration, under the condition that $\eta = 0$, $\theta = 1$, gives

$$\theta(\eta,\xi) = \frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}\left[\begin{array}{l}\frac{-1}{8\varepsilon_m Pr} ln\left(4\varepsilon_m Pr\left(\eta-\eta^2\right)+1\right) - \\ -\frac{1}{4H} Arth\left[\frac{\varepsilon_m Pr(2\eta-1)}{H}\right] - \left[\frac{1}{4H} Arth\left(\frac{\varepsilon_m Pr}{H}\right)\right]\end{array}\right] + 1, \quad (12)$$

where $H = \sqrt{\varepsilon_m Pr + \varepsilon_m^2 Pr^2}$

The temperature is determined as the bulk mean:

$$\theta_{av} = \int_0^1 \theta(\eta,\xi)\frac{u(\eta)}{\bar{u}}d\eta = \int_0^1 \frac{8}{7}\eta^{\frac{1}{7}}\theta(\eta,\xi)d\eta. \qquad (13)$$

The analytic solution of the integral (13) does not exist, but, taking into account the profile of 1/7 power, with a particular approach about the average temperature, lets replace it with a constant, then

$$\theta_{av} = \int_0^1 \theta(\eta,\xi)\frac{u(\eta)}{\bar{u}}d\eta \approx \int_0^1 \theta(\eta,\xi)d\eta. \quad (14)$$

To evaluate the correctness of replacement (13) to (14), the linear temperature profile $(1-\eta)$ which is automodeling at $\xi\rightarrow\infty$, and power-mode $\left(1-\sqrt{\eta}\right)$, which approximately corresponds to the temperature profile in the film after big wave passage may be substituted in (13) and (14) respectively. The difference in mean temperature values by (13) and (14) for the first case does not exceed 7 %, for the second – 9%.

The integration of (14) gives

$$\theta_{av}(\xi) = \frac{Pe}{4}\frac{d\theta_{av}}{d\xi}\left[\frac{1}{8\varepsilon_m Pr} - \frac{1}{4H}Arth\left(\frac{\varepsilon_m Pr}{H}\right)\left(\frac{1}{\varepsilon_m Pr}+2\right)\right] + 1. \qquad (15)$$

Marking:

$$AB = \frac{Pe}{4}\left[\frac{1}{8\varepsilon_m \mathrm{Pr}} - \frac{1}{4H}Arth\left(\frac{\varepsilon_m \mathrm{Pr}}{H}\right)\left(\frac{1}{\varepsilon_m \mathrm{Pr}}+2\right)\right]$$

and replacing (15) yields:

$$\frac{d\theta_{av}}{d\xi} = \frac{1}{B}\theta_{av} - \frac{1}{B}, \qquad (16)$$

with boundary conditions $\xi = 0$, $t(\eta, 0) = t_{sat}; \theta_{av} = 0$, which after integration gives:

$$\theta_{av} = \left[1 - exp\left(\frac{\xi}{B}\right)\right] . \qquad (17)$$

Substituting the derivative of (17) by ξ in (12), and replacing the index at ξ by ξ_m one receives a correlation for boundary curve, at which $\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=1} = 0$

$$\theta(\xi_m, \eta) = 1 - \frac{Pe}{4}\frac{1}{B} exp\left(\frac{\xi_m}{B}\right)\left[\begin{array}{l}\frac{-1}{8\varepsilon_m Pr} ln\left(4\varepsilon_m Pr\left(\eta - \eta^2\right) + 1\right) - \\ - \frac{1}{4H} Arth\left[\frac{\varepsilon_m Pr(2\eta - 1)}{H}\right] - \left[\frac{1}{4H} Arth\left(\frac{\varepsilon_m Pr}{H}\right)\right]\end{array}\right]. \qquad (18)$$

The coordinate ξ_m can be found out from (18) providing that at $\xi = \xi_m$ and $\eta = 1$; the dimensionless temperature is equal to zero, in other words $\theta \cdot (1, \xi_m) = 0$

$$\xi_m = B\, ln \frac{-8BH}{Pe\, Arth\left(\frac{\varepsilon_m Pr}{H}\right)} . \qquad (19)$$

The region $\xi \geq \xi_m$ is characterized by fact that the temperature of film surface remains constant; consequently the boundary condition (7) is changing into:

$$\eta = 0, \theta = 1; \ \eta = 1, \ \theta = 0, \qquad (20)$$

The film temperature field in the period between big waves passage $\xi \geq \xi_m$ develops from the boundary curve (18), at $\xi = \xi_m$, to the current temperature field at the distance ξ_v, at which the next mixing of the film by big wave takes place.

Double integration of (6) with the function of turbulent viscosity distribution (10) and boundary conditions (20) yields:

$$\theta(\xi, \eta) = \frac{Pe}{32}\frac{\partial\theta_{av}(\xi)}{\partial\xi}\frac{1}{\varepsilon_m Pr} ln\left(\frac{1}{1 + 4\varepsilon_m Pr\eta - 4\varepsilon_m Pr\eta^2}\right) + \frac{1}{2}\frac{Arth\left(\frac{\varepsilon_m Pr - 2\varepsilon_m Pr\eta}{H}\right)}{Arth\left(\frac{\varepsilon_m Pr}{H}\right)} + \frac{1}{2} \qquad (21)$$

The derivative $\frac{d\theta_{av}}{d\xi}$ in equation (21) will be found by the mean bulk temperature (21) considering boundary condition (18).

The average temperature of the film is:

$$\theta_{av} = \int_0^1 \theta(\eta, \xi)\frac{u(\eta)}{\bar{u}} d\eta \approx \int_0^1 \theta(\xi, \eta) d\eta = \frac{Pe}{16}\frac{\partial\theta_{av}}{\partial\xi}\frac{1}{\varepsilon_m Pr}\left[1 - \frac{(1 + \varepsilon_m Pr)}{H} Arth\left(\frac{\varepsilon_m Pr}{H}\right)\right] + \frac{1}{2} . \qquad (22)$$

Denoting

$$S = \frac{Pe}{16}\frac{1}{\varepsilon_m Pr}\left[1 - \frac{(1 + \varepsilon_m Pr)}{H} Arth\left(\frac{\varepsilon_m Pr}{H}\right)\right],$$

from (22) one receives the deferential equation

$$\frac{\partial\theta_{av}(\xi)}{\partial\xi}-\frac{1}{S}\theta_{av}(\xi)+\frac{1}{S2}=0\,, \qquad (23)$$

whose solution is:

$$\theta_{av}=\frac{1}{2}+c_2\,exp\left(\frac{\xi}{S}\right)\,. \qquad (24)$$

The integration constant c_2 will be found from the boundary condition (18) considering that at

$$\xi=\xi_m\,,\quad \theta_{av}=\theta_{av.m}\,,$$

the average temperature $\theta_{av.m}$ from the boundary curve (18) at $\xi=\xi_m$ reads:

$$\theta_{av.m}\approx\int_0^1\theta(\xi_m,\eta)d\eta=1-\frac{Pe}{4}\frac{exp\left(\frac{\xi_m}{B}\right)}{B}\left[\frac{1}{4\varepsilon_m Pr}-\right. \left.-\frac{(1+\varepsilon_m Pr)}{4\varepsilon_m Pr\,H}Arth\left(\frac{\varepsilon_m Pr}{H}\right)-\frac{1}{4H}Arth\left(\frac{\varepsilon_m Pr}{H}\right)\right] \qquad (25)$$

Substituting the received meaning $\theta_{av.m}$ in equation (24) instead of θ_{av}, we will find the integration constant c_2

$$c_2=\left[\theta_{av.m}-\frac{1}{2}\right]exp\left(-\frac{\xi_m}{S}\right),$$

Eventually, the expression for the mean temperature takes a form

$$\theta_{av}=\frac{1}{2}+\left[\theta_{av.m}-\frac{1}{2}\right]exp\left(\frac{\xi-\xi_m}{S}\right) \qquad (26)$$

Substituting the derivative of θ_{av} by ξ (21), one finally receives the expression for temperature in the film distribution in the period between big waves passage.

$$\theta(\xi,\eta)=\frac{Pe}{32}\left[\frac{\theta_{av.m}-\frac{1}{2}}{S}\right]exp\left(\frac{\xi-\xi_m}{S}\right)\frac{1}{\varepsilon_m Pr}ln\left(\frac{1}{1+4\varepsilon_m Pr\eta-4\varepsilon_m Pr\eta^2}\right)+ +\frac{1}{2}\frac{Arth\left(\frac{\varepsilon_m Pr-2\varepsilon_m Pr\eta}{H}\right)}{Arth\left(\frac{\varepsilon_m Pr}{H}\right)}+\frac{1}{2}. \qquad (27)$$

The graphical interpretation of temperature field development in the film along the heating surface between the big waves passage is shown in figure 2

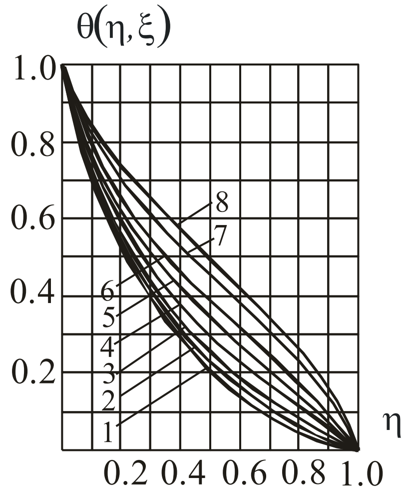

Fig. 2 The dimensionless temperature profiles between big wave passage for water by (27) in the region $\xi > \xi_m$

Re **=6780; t = 100 °C; $\varepsilon_m = 1$; $\Gamma_v = 0.5\ 10^{-3}\ \frac{m^2}{s}$.**

1 – $\xi = \xi_m = 7.3$; 2 – $\xi = 15$; 3 – 30; 4 – 60; 5 – 100; 6 – 150; 7 – 300; 8 – 1000

From (27) derivative of $\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0}$ on the boundary:

$$\left.\frac{d\theta(\xi,\eta)}{d\eta}\right|_{\eta=0} = \frac{-Pe}{8}\frac{\left(\theta_{av.m} - \frac{1}{2}\right)}{S} exp\left(\frac{\xi-\xi_m}{S}\right) - \frac{\varepsilon_m Pr}{H\left(1-\frac{\varepsilon_m^2 Pr^2}{H^2}\right) Arth\left(\frac{\varepsilon_m Pr}{H}\right)} \tag{28}$$

Heat flow on the boundary:

$$q(\xi)_{\eta=0} = -\lambda\frac{t_w - t_{sat}}{\delta}\left.\frac{d\theta}{d\eta}\right|_{\eta=0} =$$

$$= \lambda\frac{t_w - t_{sat}}{\delta}\left[\frac{Pe}{8}\frac{\left(\theta_{av.m} - \frac{1}{2}\right)}{S} exp\left(\frac{\xi-\xi_m}{S}\right) + \frac{\varepsilon_m Pr}{H\left(1-\frac{\varepsilon_m^2 Pr^2}{H^2}\right) Arth\left(\frac{\varepsilon_m Pr}{H}\right)}\right] \tag{29}$$

respectively, the mean heat flux will be found as:

$$q_{cp} = \frac{1}{\xi_v - \xi_m}\int_{\xi_m}^{\xi_v} q(\xi)_{\eta=0}\, d\xi =$$

$$= \frac{\lambda(t_w - t_{sat})}{\delta(\xi_v - \xi_m)(\varepsilon_m^2 Pr^2 - H^2)} \left\{ \begin{array}{l} \frac{Pe}{8}\left[\theta_{av.m} H^2 - \theta_{av.m}\varepsilon_m^2 Pr^2 - \frac{H^2}{2} + \frac{\varepsilon_m^2 Pr^2}{2}\right] \times \\ \times \left[1 - exp\left(\frac{\xi_v - \xi_m}{S}\right)\right] - \frac{\varepsilon_m Pr H(\xi_v - \xi_m)}{Arth\left(\frac{\varepsilon_m Pr}{H}\right)} \end{array} \right\}, \quad (30)$$

The heat transfer coefficient (HTC) is determined as $\alpha = \frac{q_{av}}{t_w - t_i}$, then from (30) one receives the expression:

$$\alpha = \frac{\lambda}{\delta(\xi_v - \xi_m)(\varepsilon_m^2 Pr^2 - H^2)} \left\{ \begin{array}{l} \frac{Pe}{8}\left[\theta_{av.m} H^2 - \theta_{av.m}\varepsilon_m^2 Pr^2 - \frac{H^2}{2} + \frac{\varepsilon_m^2 Pr^2}{2}\right] \times \\ \times \left[1 - exp\left(\frac{\xi_v - \xi_m}{S}\right)\right] - \frac{\varepsilon_m Pr H(\xi_v - \xi_m)}{Arth\left(\frac{\varepsilon_m Pr}{H}\right)} \end{array} \right\} \quad (31)$$

The length of big waves λ_{bw} respective to [18] provided that free water downflowing on the vertical pipe with a diameter of 25 mm may be approximated by the correlation

$\lambda_{bw} = 0.12 + 7.7 \cdot 10^{-6}(Re - 400)$ at $Re \geq 400$,

$$\lambda_{bw} = 0.12 \text{ at } Re \leq 400 \quad (32)$$

The thickness of turbulent film free downflowing on the vertical surface according to the data [22, 23] closely complies with the velocity distribution in the film law of 1/7 according to which:

$$\delta = 0.135\left(\frac{\nu^2}{g}\right)^{1/3} Re^{\frac{7}{12}} \quad (33)$$

at $Re \geq 2060$

At $Re \leq 2060$ according to [22, 23] the film thickness fit with the expression for laminar liquid film:

$$\delta = \sqrt[3]{\frac{3\Gamma_v \nu}{g}} \quad (34)$$

The function ε_m is determined by the comparison of HTC, calculated by (31), with experimental data with saturated turbulent and laminar liquid films in the mode of evaporation from interphase during the free downflowing.

$$\varepsilon_m = 0.15 \cdot 10^{-3}\, \text{Re}^{0.95}\left(\frac{\nu_o}{\nu}\right)^{0.45} \cdot \left[1 - \exp\left(-\frac{\text{Re}^2}{80000}\right)\right] \quad (35)$$

at $Re \leq 2060$,

$$\varepsilon_m = 0.15 \cdot 10^{-3}\, \text{Re}^{0.95}\left(\frac{\nu_o}{\nu}\right)^{0.45}\left[1 - \exp\left(-\frac{\text{Re}^2}{80000}\right)\right] + $$
$$+ 0.4 \cdot 10^{-3} \exp\left[\frac{1}{0.63}\left(1 - \frac{\nu}{\nu_o}\right)\right](\text{Re} - 2060)^{0.83}$$

at $Re \geq 2060$, where $\nu_o = 0.3 \cdot 10^{-6}, \frac{m^2}{s}$.

Graphical interpretation of correlation (35) for sugar solutions and water is shown at figure 3.

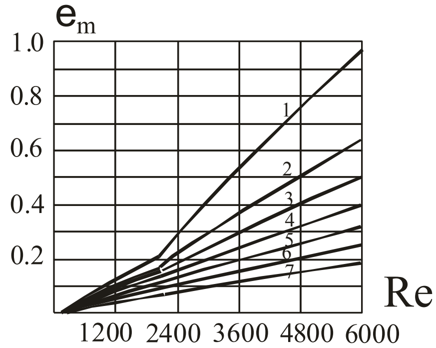

Fig. 3 Correlation $\varepsilon_m = f(Re)$ for sugar solutions and water at $t = 100\,^oC$
1 – water; 2 – 7- sugar solution, 2 – DM = 20%; 3 – 30%; 4 – 40%; 5 – 50%; 6 – 60%; 7 – 70%

A comparison of experimental data on heat transfer to sugar solutions and water films downflowing on the vertical surface to those calculated for the evaporation regime with interphase is shown in figure 4.

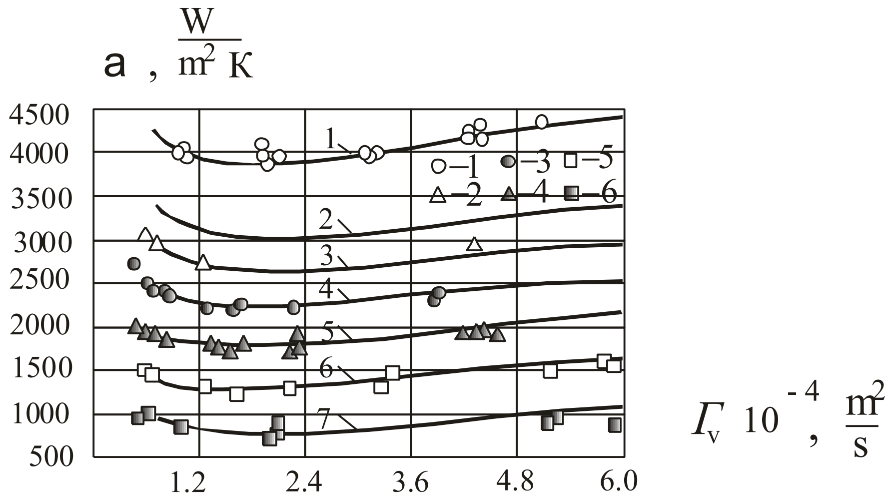

Fig.4 A comparison of experimental data on heat transfer to sugar solutions and water films downflowing on the vertical surface to those calculated for the evaporation regime with interphase
The lines correspond to the calculations by correlations (31, 32, 33)
1 – water, t = 100 C; 2- 7 – sugar solution, 2 – DM = 20%; 3 – 30%; 4 – 40%; 5 – 50%; 6 – 60%; 7 – 70%.
Points: 1 – water, t = 100 C; 2- 6 – sugar solution, 2 – DM = 30%; 3 – 40%; 4 – 50%; 5 – 60%; 6 – 70%

A turbulent film thickness in the presence of interphase shear stress may be determined from the momentum conservation equation.

$$\frac{\tau_i \delta}{\rho \nu} + \frac{g\delta^2}{\nu}(1-\eta) = \left[1 + \left(\frac{\nu_t}{\nu}\right)\right]\frac{du}{d\eta}, \quad (36)$$

Using the M.D.Millionschicov expression (10) for the turbulent viscosity, in which, according to [23], the constant coefficient 0.39 is changed to a function.

$$m = 0.328\left[1 - 2.3 exp\left(-5 \cdot 10^{-4}\, Re^{0.98}\right)\right]$$

at $Re \geq 1940$ (37)

$m = 0$ at $Re \leq 1940$ in the laminar film layer, the velocity profile can be found as: $u_l(\eta) = \left(\frac{\tau_i \delta}{\rho \nu} + \frac{g\delta^2}{\nu}\right)\eta - \frac{g\delta^2}{\nu}\frac{\eta^2}{2}$, (38) and in the turbulent regime:

$$u_t(\eta) = \left[\frac{2}{N}\left(\frac{\tau_i \delta}{\rho\nu} + \frac{g\delta^2}{\nu}\right) - \frac{g\delta^2}{N\nu}\left(1 + \frac{\delta_l^+}{\delta^+}\right)\right]\left\{artcg\left[\frac{m}{N}\left(2\eta_l\delta^+ - \delta^+ - \delta_l^+\right)\right] - arctg\left[\frac{m\left(2\eta\delta^+ - \delta^+ - \delta_l^+\right)}{N}\right]\right\} +$$

$$+\left(\frac{g\delta^2}{\nu}\right)\left\{\frac{1}{2\delta^+ m} ln\left|\frac{m\delta^+\left(\eta^2 - \eta\right) - m\delta_l^+(\eta - 1) - 1}{m\delta^+\left(\eta_l^2 - \eta_l\right) - m\delta_l^+(\eta_l - 1) - 1}\right|\right\} + \left(\frac{\tau_i\delta}{\rho\nu} + \frac{g\delta^2}{\nu}\right)\eta_l - \frac{g\delta^2}{\nu}\frac{\eta_l^2}{2},$$

(39)

$$N = \sqrt{2m^2\delta^+\delta_l^+ - m^2\delta^{+2} - m^2\delta_l^{+2} - 4m\,\delta^+}\ .$$

Having calculated the average velocity of the liquid in the film:

$$\bar{u} = U_l\eta_l + U_t(1 - \eta_l) =$$

$$= \int_0^{\eta_l} u_l(\eta)d\eta + \int_{\eta_l}^{1} u_t(\eta)d\eta \qquad (40)$$

where $U_l = \frac{1}{\eta_l}\int_0^{\eta_l} u_l(\eta)d\eta$;

$U_t = \frac{1}{1 - \eta_l}\int_{\eta_l}^{1} u_t(\eta)d\eta$ – average velocities in the laminar and turbulent layers respectively, the thickness of the turbulent film in the presence of interphase friction is received, as $\delta = \Gamma_v / \bar{u}$.

A comparison of the calculated water film thickness with the experimental data, depending on the air velocity in the tube with the diameter of 25 mm in the mode of developed turbulence is shown in the figure 5.

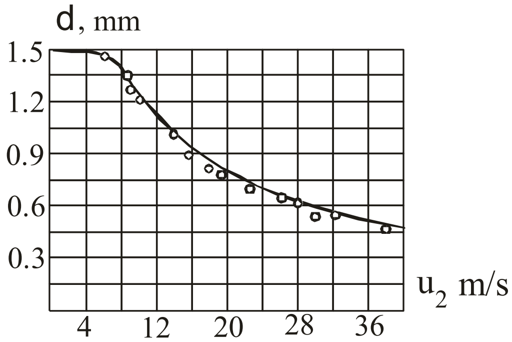

Fig.5 A comparison of the calculated water film thickness with the experimental data, depending on the air velocity in the tube with the diameter of 25 mm in the mode of developed turbulence. Points –it is data [21] water with the temperature of 25°C. d = 25 mm. Water consumption $\Gamma_v = 3 \cdot 10^{-4}$ m²/s. Re = 13200. The line – is a correlation (37, 38, 39, 40)

The function ε_m in the case of interphase friction existence differs from (35) and is determined as $\varepsilon_m E_u$. The type of multiplier to (35) E_u is determined by a correlation of the measured results of heat transfer intensity in turbulent flows with the interphase friction to the calculated by (31,32,33,37,40). Obtained thus formula for E_u reads:

$$E_u = -0.8\left(\frac{u_2}{15} - 0.5\right)^2 + 0.08\left(\frac{u_2}{15} - 0.5\right) + 1.24 \text{ at } u_2 \le 14$$

$$E_u = 1.124 - 72\left[We_2^{0.2} - \left(\frac{\rho_2 14^2}{\sqrt{\sigma g \rho_2}}\right)^{0.2}\right]^{0.7} Re^{-0.6}\left(\frac{\nu}{\nu}\right)^{0.6} \text{ at } u_2 \ge 14, \qquad (41)$$

In the case of $E_u \le 0$, E_u=0.

Graphical interpretation of the correlation (41) is given in the figure 6

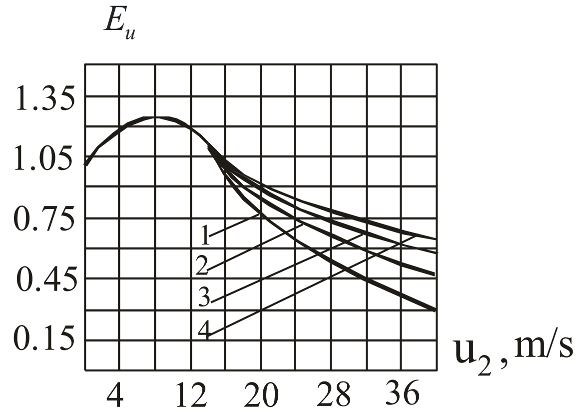

Fig. 6 $E_u = f(u_2)$ **correlation for water at** $t = 100\,^oC$.
1 – Γ_v **= 0.2 10^{-3} ,m^2/s; ; 2 – 0.3 10^{-3}; 3 – 0.4 10^{-3}; 4 – 0.5 10^{-3}**

In case of the co-current steam flow a shear stress appears on the interface, which is determined as

$$\tau_i = \zeta \rho_2 \frac{u_2^2}{8},$$

The value of interface friction coefficient can be determined from [11]. Suppression of turbulent intensity in the film with the increasing of steam velocity above 14 m/s is explained by the significant film thickness falling as a result of interphase friction, figure 5.

A comparison of the experimental data of HTC to sugar solutions with concentrations up to 70% ($\nu = 3.77 \cdot 10^{-6}\ m^2/s$) and to water at the film downflowing on the vertical surface with the interphase shear stress with correlations (31,32,35,37,40,41) are shown in the figure 7.

As it can be seen from the given plots, the correlations (31, 32, 35, 37, 40, 41) are also valid within not only the turbulent bat in viscous laminar range.

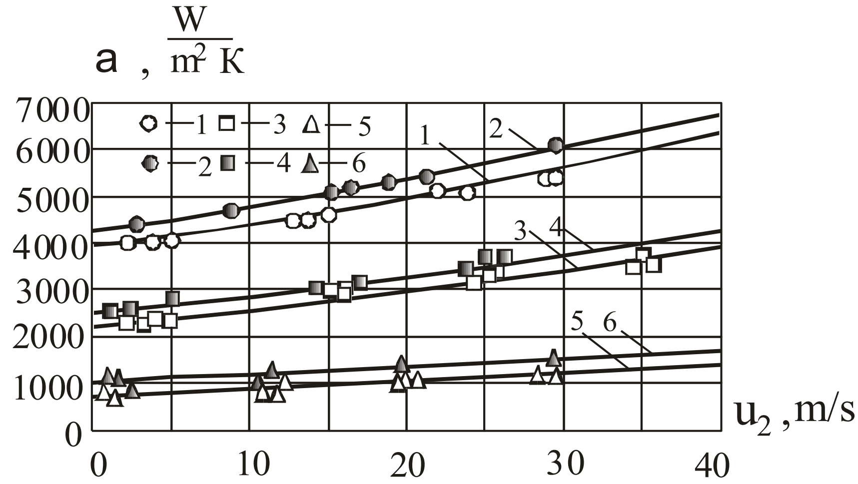

Fig. Effect of steam flow on the HTC $\alpha = f(u_2)$ for sugar solutions and water at t = 100 °C
1,2 − water; 3, 4 – sugar solution, DM = 40%; 5, 6 – sugar solution, DM = 70%; 1, 3, 5 – Γ_v = 0.3 10^{-3} ,m^2/s; 2, 4, 6 – Γ_v = 0.5 10^{-3} ,m^2/s.
Lines – a calculation for (31, 32, 35, 37, 40, 41)

4. Conclusions

1. Big waves downflowing in the vertical channels that have a powerful central whirl which in turn periodically mixes the bulk of the film exerts a main impact on the heat transfer.
2. At the presence of big surficial waves it is advisable to utilize a developed simplified turbulence model (10) equation, which is a modification of M.D.Millionschykov model [20]. Thus developed model being applied, allows obtaining an analytical solution of the heat transport differential equations and eventually obtaining simplified integral process characteristics.
3. The equations (30, 31, 34, 36, 39, 40) are recommended to be used for the calculation of HTC to the water and sugar solutions (up to 70% DM) at a regime of free falling and the presence of co-current steam flow, either. The equations may be applied to the films at a regime of evaporation from the interface of laminar and turbulent films within the marked above range of flow rates.

5. References

[1]. GIMBUTIS G., Heat Transfer in a Liquid Film Gravitational Flow, *Vilnius:* 233, (1988)
[2]. VORONTSOV.E.G., TANANAYKO Y.M., Heat Transfer in Liquid Films, *Technique K.*: 196 (1972)
[3]. TANANAYKO Y.M., VORONTSOV E.G., Methods of Calculation and Research of Film Processes, *Technique K.:* 312 (1975)
[4]. RIABCHUK O.M., Thermal and Hydrodynamic Processes Evaporation in Downstream Film Flows of Sugar Solutions, Dissertation, *Kyiv*: 167 (2013)
[5]. SOBIN V.M., Heat and Mass Transfer in the Film Flows at Complicated Conditions. *MH Science and Technology:* 303 (1994)
[6]. VORONTSOV E.G., Hydrodynamics and Heat Exchange in the Watering Chemical Films Equipment, *Ph.d Dissertation. MM: Inorganic Chemistry, Academy of Sciences USSR:* (1989)
[7]. VORONTSOV E.G. Research of Average Wave Characteristics in the Calculation of Heat and Mass Transfer in Films. Heat and Mass Transfer, *MMF-92. Minsk,* t.2: 43 (1992)
[8]. VORONTSOV E.G., Untersuchung mittelstatistischer Charakteristiken von Wellenströmungen in Rieselfilmen, *Brennstoff Wärm Kraft*, Bd.42. No12: 730, (1990)
[9]. VORONTSOV E.G., The Thermal Diffusivity of Falling Films. TOChT, Vol. 33, No 2: 117- 127, (1999)

[10]. GANCHEV B. G., Cooling of Nuclear Reactor of Elements by Film Flow, *Energoatomizdat, Moscow:* 192 (1987)
[11]. PRYADKO M.O., GLOBA O.V., FORSYUK A.V., GLOBA V.Z., Film Flows in Tubes of Apple Juice Evaporators, *SCIENTIFIC WORKS of National University of Food Technologies*, Vol. 21, No 2: 88 – 95, (2015)
[12]. DEMEKHIN E.A., KALAIDIN E.H., RASTATURIN A.A., Effect of Wavy Regimes Upon the Mass Transfer in Down Flowing Liquid Film, *Termophysics and aerodynamics,* Vol. 12, No 2: 259-269, (2005)
[13]. PETRENKO V.P., ZASYADKO Y.I, Heat Transfer Modeling Downflowing Laminar Films the Developed Wavy Structure With Co-Current Steam Flow, *Food and Environment Safety*, Vol.15, Issue. 3: 203-215, (2016)
[14]. JEPSEN J.C.,CROSSER O.K.,PERRY R.H. The effect of wave induced turbulence on the rate of absorption of gases in falling film. Am.Inst.Chem. Engng. Journal, (1966), Vol.12, No 1, pp.185 – 192.
[15]. LAMOURELLE A.P., SANDAL O.C.Gas absorption into turbulent liquid. Chem.Engn.Science. (1972). Vol. 27, pp.1035–1043.
[16]. GANCHEV B.G., KOZLOV V.M. Experimental research of a hydrodynamic liquid film structure with the free downflowing on a vertical surface. Tohti, (1973). V.7, No 5, pp. 727 - 733.
[17]. GROSSMAN G., HEATH M. Simultaneous heat and mass transfer in absorption of gases in turbulent liquid films. Int. J. Heat Mass Transfer. (1984). Vol. 27, No 12, pp. 2365 – 2376.
[18]. VAN DRIEST E.R. On turbulent flow near a wall. J. Aeronaut. Sci. (1956), Vol. 23, pp. 1007– 1011.
[19]. REICHARDT H.,ANGEV Z. Math. Mech. 31, 208 (1951); W. M. Kays, Convective Heat and Mass Transfer, p.71. McGraw–Hill, New York (1966).
[20]. MILLIONSCHYKOV M.D. Basic laws of turbulent flow in the boundary layer. – Nuclear power, (1970), Vol. 28, Issue. 4, pp. 317 – 320.
[21]. KULOV N. N., Hydrodynamics and a mass exchange in the film and disperse streams descending, Dissertation, Moscow, (1984), p.409
[22]. PETRENKO V.P., PRYADKO M.O., RIABCHUK O.M Film thickness in descending circular flow with interfacial interaction. Labor NUFT , (2016), Vol. 22, No 3, pp. 150 -156.