



## HEAT TRANSFER MODELING IN DOWN FLOWING LAMINAR FILMS WITH THE DEVELOPED WAVY STRUCTURE WITH CO-CURRENT STEAM FLOW

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**Abstract:** *The result of heat transfer modeling in down flowing viscous laminar films with the developed wavy structure at the regime of evaporation from film interface at free falling and with co-current steam flow, a heat transfer model, which takes into account a cyclic process of temperature field relaxation with the periodic mixing of film by big waves, has been presented. A mathematical heat transfer model in laminar saturate liquid film with the developed wavy structure on its surface has been developed. The model takes into consideration cyclic relaxation of transient temperature field which happens right after the passage of a powerful big wave. The developed mathematical model describes the time history of the two dimensional temperature fields as a function of the Peclet number and the core characteristic of the wavy motion (the length of big waves). Based upon the proposed model a set of correlations has been obtained. These ones are proposed as a means for the generalization of heat transfer experimental data, obtained within the experimental studies of liquid films, heated to the saturation temperatures and evaporation from the interface. A generalized equation has been derived, which can be used for the calculations of Heat Transfer Coefficients (HTC) to the saturated sugar solutions liquid films. This equation contains wavy characteristics of down flowing films and are valid within the range of parameters characteristic of sugar industry evaporators, namely: concentrations – 0...70 % dry matter; liquid mass flow rate density– $0.01 \times 10^3 \dots 0.6 \times 10^{-3} \text{ m}^2/\text{sec}$ , the Peclet number range – 400...25000.*

**Key words:** *films, waves, heat transfer, temperature, evaporation.*

### 1. Introduction.

A structure of liquid films down flowing over the vertical tubular surface of a film evaporator changes along the length of a channel. This happens as a result of a continuous development of wave structure and due to the growth of the co-current steam velocity. Therefore, the

experimental results of studies on the heat and hydrodynamic processes at models with different channel geometry may differ significantly. The predominant amount of theoretical papers related to the wavy movement of films [1, 2, 3] is dedicated to the research into the development of regular waves structure. The part of

such waves on the full length of heat transfer tubes of industrial evaporators is negligibly small if compared to the regions of big waves. In the available papers dedicated to the research of heat transfer in down flowing films the effect of wavy structure on heat transfer has been taken into account indirectly through the parameters of turbulence and film interface shear stress.

Predominant number of experimental works dedicated to the research of heat transfer to the films, heated to the saturation temperature, has been obtained in the experimental units modeling industrial evaporators [4, 5, 6]. Thus, the experimental data obtained reflect the effect of number of factors, which influence simultaneously heat transfer, without separation of components.

Therefore, the experimental correlations thus obtained, which are recommended for the calculations of the heat transfer to the saturated films in industrial evaporators are limited in terms of the applicable ranges of regime parameters similar to the model ones, as well as in terms of the geometry of the experimental unit. Keeping in mind the statements mentioned above the development of adequate physical model of film flows relevant to a concrete structural form of films looks of current interest.

According to [7], in case of free falling films over the vertical surface the saturation of wavy movement with fully developed big wave structure occurs at a distance approximately 2...2.5 m from the film forming device. These big waves move on the film interface with the phase velocity, which 1.5...2 times meant film velocity [8]. An important factor, which affects seriously hydrodynamic film structure, is the fact that big waves contain a central swirl. Due to this the liquid in the bulk of film is periodically mixed, which in turn causes the velocity, temperature and concentration profiles deformation. It

should be mentioned also that in tubular channels of 7...12 m long industrial film evaporators there is a continuous growth of co-current steam core due to the gradual film evaporation. This, in turn, causes a continuous growth of interface shear stress, which distorts wavy and hydrodynamic structure of film flow. Taking into account all mentioned above, the heat transfer in liquid films may be treated as a cyclic process of temperature field relaxation, which happens after each consequent passage of a big wave. This big wave due to the existence of a powerful central swirl, which transports slightly, superheated liquid from the boundary region to the external side of film. According to this mechanism the peak of a big wave will be constantly fed with the slightly superheated liquid and transfer it to the wave surface, from which evaporation takes place, thus transferring heat to the steam core. It is clear that the more powerful the wave, the bigger the deformation of temperature profile in the vicinity of heated surface right after the wave is. Within the periods between the big waves passage a relation of temperature field takes place. The process of relaxation is stretched along the film movement until a consecutive mixture of liquid happens due to the passage of the consecutive wave.

## **2. Materials and methods**

A direct experiment of heat transfer in down flowing liquid films heated to the saturation temperatures with sugar solutions as model liquids has been carried out at the experimental unit with the independent formation of phases' mass flow rates and heat flux. The main core of the experimental unit was represented by a stainless still pipe

with the inside diameter of 20 mm and 1.8 m long. The experimental tube was separated into the initial 1.5 m stabilization section and 0.3 m measurement section. The down flowing of water (sugar solutions) film has been formed by means of overflowing over the tube's upper rim. In the event of steam-liquid flow modeling, dry saturated steam has been supplied in co-current regime. The liquid falling film has been heated by dry saturated steam which was supplied into outside heating sections attached to the experimental tube. The heating chambers were designed in such a way as to provide an individual heating of the stabilization section and the experimental one. The mentioned sections

were hooked up to the individual vacuum-condensation sections which allowed for the keeping of different pressures in each chamber. Such arrangement allowed also the maintaining of vacuum down to 0.8 bars and thus, to vary the temperature head between the heating steam temperature and evaporation temperature. Special probes for taking samples of liquid to determine its concentration and measurements of temperatures were positioned directly after the measurement section. A detailed description of the experimental unit is given in [11] and its schematic is given in fig.1.

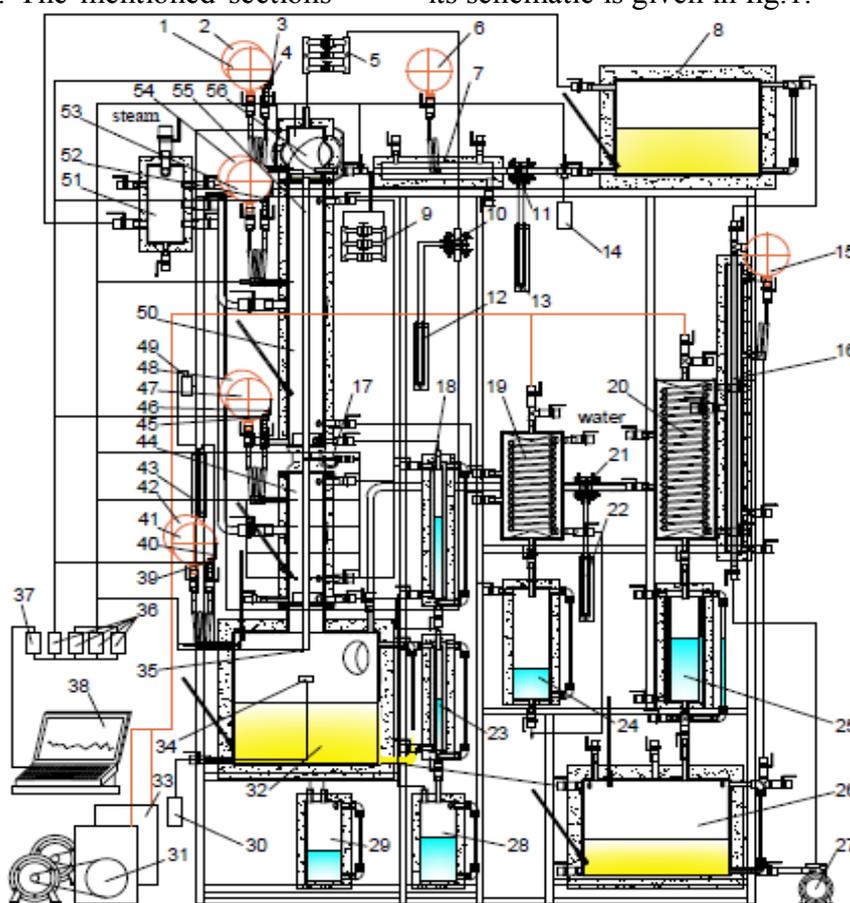


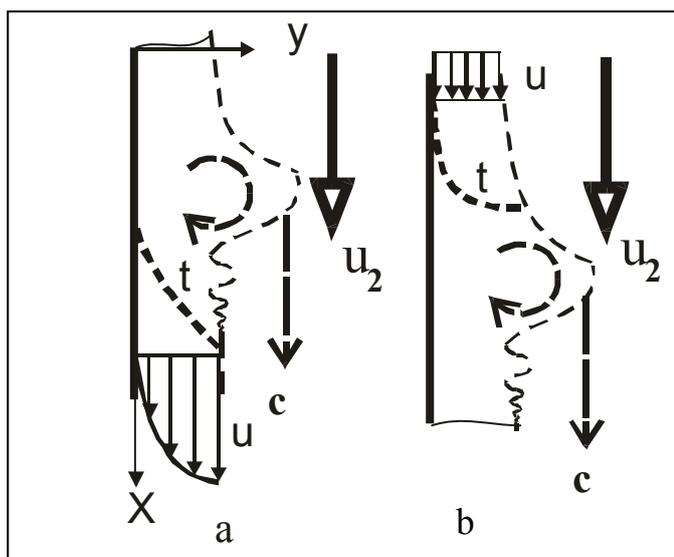
Figure 1. Scheme of the experimental stand

1,6,15,41,47,53 – manometer; 2,42,48,54 – vacuum manometer; 3,39,45 – sensor for vacuum control; 4,40,46,52 – sensor for pressure control; 5,9 – collector flow control; 7,16 – heat exchanger; 8,26,32 – tank; 10,11,21 – diaphragm; 12,13,22,43 – differential pressure gage; 14,30 – sampler; 17 – gage probe; 18,23,24,25,28,29 – hotwell; 19,20 – condenser; 27 – pump; 31,33 – vacuum pump; 34 – tank for sampling; 35 – tank of solution's temperature measurement on the exit of pipe; 36 – module analog input 7018P; 37 – module 7520; 38 – computer; 44,50 – heating chamber; 49 – equaling tank; 51 – separator of heating steam; 55 – experimental pipe; 56 – top chamber.

### 3. Results and Discussion

According to the proposed heat transfer model the rolling of a big wave along the film surface causes local mixing of liquid and deformation of velocity and temperature fields and equalizing the field of concentrations, respectively. It could be suggested that right prior to the passage of a big wave

a velocity profile is close to the parabolic one, although to a certain extent deformed due to the interface friction of the steam core. The temperature profile close to the linear is shown in Fig. 2a. Right after a big wave passage, velocity becomes constant across the film thickness, whereas the temperature profile will be curved, Fig. 2b.



**Figure 2. Schematic of big wave passage on the film surface**  
**a) velocity and temperature profiles prior to big wave passage; b) the same – afterwards**

Since the amplitude of big waves is insignificant, the distortion of temperature curve right behind the wave could be not deep and  $\frac{\partial \theta}{\partial \eta_{\eta=1}} \geq 0$ . In this

case the evaporation from the interface right behind the wave reaches its minimum value and, further on, its growth. In the event of big wave amplitudes and powerful swirls inside the bulk of the film, respectively, there could be a noticeable transfer of unheated liquid in the direction towards the wall. Under this condition there would be cessation of evaporation until the moment temperature gradient appears on the surface. Thus, a

maximum heat flux on the wall will occur in the moment of a big wave passage, when temperature of the boundary layer will reach its maximum value. Further on in the process of film heating up the wall flux will decrease and that one on the interface will grow, asymptotically approaching the value, at which temperature profile becomes linear.

In light of the above, a process of temperature field relaxation could be split into two periods:

- the first one lasts until the temperature gradient on the interface appears,
- the second one – after that.

It is clear that the first period starts right after the passage of a big wave, Fig. 2b. The liquid in film is mixed at a constant temperature; velocity is constant across the thickness

$$\bar{u} = \frac{1}{\delta} \int_0^\delta \left[ \left( \frac{\tau_i}{\rho\nu} + \frac{g\delta}{\nu} \right) y - \frac{g}{2\nu} y^2 \right] dy = \frac{\tau_i\delta}{2\rho\nu} + \frac{g\delta^2}{3\nu}, \quad (1)$$

where  $u = \left( \frac{\tau_i}{\rho\nu} + \frac{g\delta}{\nu} \right) y - \frac{g}{2\nu} y^2$  – velocity distribution across laminar field

$$\left[ \frac{\tau_i\delta}{2\rho\nu} + \frac{g\delta^2}{3\nu} \right] \delta \frac{\partial\theta(\eta, \xi)}{\partial\xi} = a \frac{\partial^2\theta(\eta, \xi)}{\partial\eta^2}, \quad (2)$$

where  $\theta(\eta, \xi) = \frac{t(\eta, \xi) - t_{sat}}{t_w - t_{sat}}$  – dimensionless temperature;  $\eta = \frac{y}{\delta}$ ,

$\xi = \frac{x}{\delta}$  – dimensionless transverse and longitudinal coordinates;  $\delta$  – film thickness;  $\Gamma_v$  – volumetric liquid flux;

$a$  – temperature conductivity;  $\nu$  – cinematic viscosity coefficient;  $\tau_i$  – shear stress on the film interface;  $t_{sat}$  – saturation temperature;  $t_w$  – wall temperature.

Since the film thickness and mean fluid velocity are interrelated as a cubic equation

$$\Gamma_v = \bar{u}\delta = \frac{\tau_i\delta^2}{2\rho\nu} + \frac{g\delta^3}{3\nu}, \quad (3)$$

then (2) we read

$$\frac{Pe}{4} \frac{\partial\theta(\eta, \xi)}{\partial\xi} = \frac{\partial^2\theta(\eta, \xi)}{\partial\eta^2}, \quad (4)$$

where  $Pe = \frac{4\Gamma_v}{a} = \frac{4\bar{u}\delta}{a}$  – the Peclet number.

$$\left[ \left( \frac{\tau_i}{\nu\rho\delta} + \frac{g}{\nu} \right) \eta - \frac{g}{2\nu} \eta^2 \right] \delta^3 \frac{\partial\theta(\eta, \xi)}{\partial\xi} = a \frac{\partial^2\theta(\eta, \xi)}{\partial\eta^2}. \quad (7)$$

Having substituted the left hand side of (7) by the mean integral velocity,

of the film, the presence of interface friction the velocity in the film could be determined as the mean integral across the film

with interface friction due to the steam core movement.

Within this period the heat transfer equation is as follows

The solution of (4) with the boundary conditions

$$\begin{aligned} \theta(0,0) &= 0, \quad \theta(0,\xi) = 1, \\ \theta(1,0) &= 0, \quad \frac{\partial\theta(\infty,\xi)}{\partial\eta} = 0, \end{aligned} \quad (5)$$

would be

$$\theta(\eta, \xi) = \operatorname{erfc} \left( \frac{\eta}{4} \sqrt{\frac{Pe}{\xi}} \right). \quad (6)$$

The profile (6) will be developed until the temperature gradient at a distance  $\xi_m$  appears on the interface. Further on, the temperature on the film surface will remain constant as a result of evaporation. The limiting temperature curve, which signifies a change of temperature field ( $\frac{\partial\theta}{\partial\eta}|_{\eta=1} = 0$ )

can be determined from the heat transfer equation with the parabolic velocity profile. It should be kept in mind that within the process of film heating up its velocity will change

$$\int_0^1 \left[ \left( \frac{\tau_i}{\rho \delta \nu} + \frac{g}{\nu} \right) \eta - \frac{g}{2\nu} \eta^2 \right] \frac{\delta^3}{a} \frac{\partial \theta(\eta, \xi)}{\partial \xi} d\eta = \left( \frac{\tau_i \delta^2}{2\rho \nu} + \frac{g \delta^3}{3\nu} \right) \frac{1}{a} \frac{\partial \theta_{av}(\xi)}{\partial \xi}$$

and combining (3), one obtains

(7) as

$$\frac{Pe}{4} \frac{\partial \theta_{av}(\xi)}{\partial \xi} = \frac{\partial^2 \theta(\eta, \xi)}{\partial \eta^2}. \quad (8)$$

Double integration of (8) with the boundary conditions

$$\begin{aligned} \eta = 0, \quad \theta = 1; \\ \eta = 1, \quad \frac{\partial \theta}{\partial \eta} = 0, \end{aligned} \quad (9)$$

yields

$$\theta_{av} = \int_0^1 \theta(\eta, \xi) \frac{u(\eta)}{\bar{u}} d\eta = \int_0^1 \theta(\eta, \xi) \frac{\left[ \left( \frac{\tau_i}{\rho \nu} + \frac{g \delta}{\nu} \right) \eta - \frac{g \delta}{2\nu} \eta^2 \right]}{\left( \frac{\tau_i}{2\rho \nu} + \frac{g \delta}{3\nu} \right)} d\eta. \quad (11)$$

Integrating (11) one obtains

$$\theta_{av} = 1 - \frac{\partial \theta_{av}(\xi)}{\partial \xi} \frac{Pe}{80} \frac{25\tau_i + 16\rho g \delta}{3\tau_i + 2\rho g \delta}. \quad (12)$$

$$\text{Denoting } D = \frac{Pe}{80} \frac{25\tau_i + 16\rho g \delta}{3\tau_i + 2\rho g \delta},$$

and substituting in (12) yields first order differential equation

$$\theta(\eta, \xi_m) = \left( \frac{Pe}{4} \right) \frac{1}{D} \exp\left( -\frac{\xi_m}{D} \right) \left( \frac{\eta^2}{2} - \eta \right) + 1. \quad (15)$$

The coordinate  $\xi_m$  may be denominated from (15) at a suggestion that at  $\xi = \xi_m$  the dimensionless temperature equals 0,  $\theta(1, \xi_m) = 0$

$$\xi_m = D \ln\left( \frac{Pe}{8D} \right). \quad (16)$$

Beginning with  $\xi \geq \xi_m$ , the second period of the temperature field development starts. During the whole second period the film surface temperature remains constant to the fact

$$\theta(\eta, \xi) = \frac{Pe}{8} \frac{\partial \theta_{av}(\xi)}{\partial \xi} (\eta^2 - \eta) - \eta + 1. \quad (18)$$

$$\theta(\eta, \xi) = \frac{Pe}{4} \frac{\partial \theta_{av}(\xi)}{\partial \xi} \left( \frac{\eta^2}{2} - \eta \right) + 1. \quad (10)$$

A derivative  $\frac{\partial \theta_{av}(\xi)}{\partial \xi}$  may be found from the equation for mean film temperature

$$\frac{\partial \theta_{av}}{\partial \xi} + \theta_{av} \frac{1}{D} = \frac{1}{D}, \quad (13)$$

which can be solved with the following boundary conditions:  $\xi = 0, \theta_{av} = 0,$

$$\theta_{av} = 1 - \exp\left( -\frac{\xi}{D} \right). \quad (14)$$

Substituting the derivative of (14) into (10) one obtains a temperature distribution in the film at  $\xi = \xi_m$ :

that evaporation from the surface goes on. Within this period, the boundary conditions (9) change into

$$\begin{aligned} \eta = 0, \quad \theta = 1; \quad \eta = 1, \quad \theta = 0; \\ \xi = \xi_m, \end{aligned} \quad (17)$$

$$\theta(\eta, \xi_m) = \left( \frac{Pe}{4} \right) \frac{1}{D} \exp\left( -\frac{\xi_m}{D} \right) \left( \frac{\eta^2}{2} - \eta \right) + 1$$

Integration of (8) with the boundary condition (17) yields:

The derivative  $\frac{\partial \theta_{av}(\xi)}{\partial \xi}$  will be found by the determination of the film

mean temperature at a section  $\xi \geq \xi_m$ , which gives

$$\theta_{av} = \int_0^1 \left( \frac{Pe}{8} \frac{\partial \theta_{av}}{\partial \xi} (\eta^2 - \eta) - \eta + 1 \right) \frac{\left[ \left( \frac{\tau_i}{\rho\nu} + \frac{g\delta}{\nu} \right) \eta - \frac{g\delta}{2\nu} \eta^2 \right]}{\left( \frac{\tau_i}{2\rho\nu} + \frac{g\delta}{3\nu} \right)} d\eta. \quad (19)$$

Integration of (19) yields

$$\theta_{av} = \frac{\tau_i + \frac{3}{4} \rho g \delta}{3\tau_i + 2\rho g \delta} - \frac{\partial \theta_{cp}(\xi)}{\partial \xi} \frac{Pe}{160} \frac{(7\rho g \delta + 10\tau_i)}{(3\tau_i + 2\rho g \delta)}. \quad (20)$$

Having introduced the following  $R = \frac{(7\rho g \delta + 10\tau_i) Pe}{(3\tau_i + 2\rho g \delta) 160}$ ,

which has its solution

$$\theta_{av} = R_1 + C_0 \exp\left(-\frac{\xi}{R}\right). \quad (22)$$

$$R_o = \frac{4\tau_i + 3\rho g \delta}{7\rho g \delta + 10\tau_i} \frac{40}{Pe},$$

The constant of integration  $C_0$  is determined through the films mean temperature at a moment when a temperature field starts development at  $\xi = \xi_m$  with boundary condition (17)

$$R_1 = RR_o = \frac{\tau_i + \frac{3}{4} \rho g \delta}{3\tau_i + 2\rho g \delta},$$

a differential equation is obtained

$$\frac{\partial \theta_{av}}{\partial \xi} + \theta_{av} \frac{1}{R} = R_o, \quad (21)$$

$$\theta_{av.m} = \int_0^1 \left[ \frac{Pe}{4} \frac{1}{D} \exp\left(-\frac{\xi_m}{D}\right) \left( \frac{\eta^2}{2} - \eta \right) + 1 \right] \frac{\left[ \left( \frac{\tau_i}{\rho\nu} + \frac{g\delta}{\nu} \right) \eta - \frac{g\delta}{2\nu} \eta^2 \right]}{\left( \frac{\tau_i}{2\rho\nu} + \frac{g\delta}{3\nu} \right)} d\eta. \quad (23)$$

Further integration of (23) yields

Now substituting  $\theta_{av.m}$  in (22), the integration constant  $C_0$  is

$$\theta_{av.m} = 1 - \frac{Pe}{80} \frac{(16\rho g \delta + 25\tau_i)}{D(3\tau_i + 2\rho g \delta)} \exp\left(-\frac{\xi_m}{D}\right)$$

$$C_0 = \left[ 1 - R_1 - \frac{Pe}{D80} \frac{16\rho g \delta + 25\tau_i}{3\tau_i + 2\rho g \delta} \exp\left(-\frac{\xi_m}{D}\right) \right] \exp\left(\frac{\xi_m}{R}\right),$$

and respectively, film's mean temperature in the region  $\xi \geq \xi_m$  will be

$$\theta_{av} = R_1 + \left[ 1 - R_1 - \frac{Pe}{D80} \frac{16\rho g \delta + 25\tau_i}{3\tau_i + 2\rho g \delta} \exp\left(-\frac{\xi_m}{D}\right) \right] \exp\left(\frac{\xi_m - \xi}{R}\right). \quad (24)$$

Differentiating of (24) by  $\xi$  yields

$$\frac{\partial \theta_{av}}{\partial \xi} = \left[ \frac{R_1}{R} - \frac{1}{R} + \frac{Pe}{DR} \frac{16\rho g \delta + 25\tau_i}{80(3\tau_i + 2\rho g \delta)} \exp\left(-\frac{\xi_m}{D}\right) \right] \exp\left(\frac{\xi_m - \xi}{R}\right). \quad (25)$$

After substitution of the derivative (25) into the initial equation (18) we obtain the temperature

$$\theta(\eta, \xi) = \frac{Pe}{8} \left[ \frac{R_1 - 1}{R} + \frac{Pe}{80DR} \frac{(16\rho g \delta + 25\tau_i)}{(3\tau_i + 2\rho g \delta)} \exp\left(-\frac{\xi_m}{D}\right) \right] \exp\left[\frac{\xi_m - \xi}{R}\right] (\eta^2 - \eta) - \eta + 1. \quad (26)$$

Graphic interpretation of (26) within  $\xi \geq \xi_m$ , and (6) at  $\xi \leq \xi_m$ , and (15) at  $\xi = \xi_m$  is given in Fig. 3.

The bigger the interface shear stress  $\tau_i$ , the thinner the film will be.

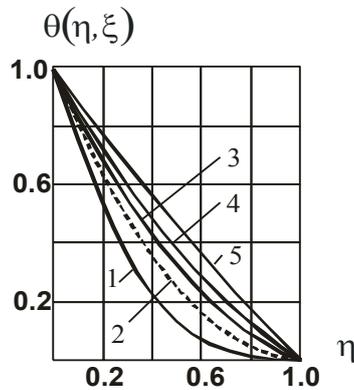


Figure 3. Temperature distribution in water film

Calculated by (6, 15, 26) at  $t = 100^\circ\text{C}$ ;  $\tau_i = 0$ ;

$$\Gamma_v = 0.5 \cdot 10^{-3} \frac{\text{m}^2}{\text{s}}, \quad \xi_m = 267;$$

1 –  $\xi = 167$ , equation(6);

2 –  $\xi = \xi_m$ , equation(15);

3 –  $\xi = 367$ ; 4 –  $\xi = 467$ ;

5 –  $\xi = 767$ ; 3, 4, 5 – equation (26).

A relative distance  $\xi$  will be bigger at the same mass flow rate, and

$$\frac{d\theta(\eta, \xi)}{d\eta_{\eta=0}} = -\frac{Pe}{8} \left[ \frac{R_1 - 1}{R} + \frac{Pe}{80DR} \frac{(16\rho g \delta + 25\tau_i)}{(3\tau_i + 2\rho g \delta)} \exp\left(-\frac{\xi_m}{D}\right) \right] \exp\left[\frac{\xi_m - \xi}{R}\right] - 1. \quad (27)$$

distribution in the film in the region  $\xi \geq \xi_m$

distance between the peaks of waves. Therefore, linearization of temperature profile will happen much quicker in the presence of interface shear stress, then in the case of film free fall, Fig. 4.

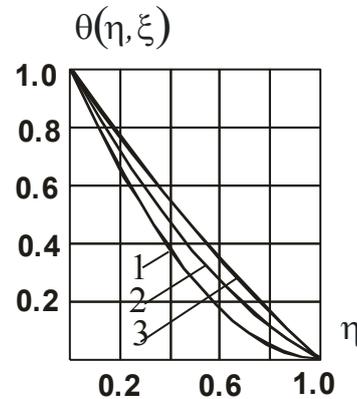


Figure 4. Temperature distribution in water film

Calculated by (6, 15, 26) at  $t = 100^\circ\text{C}$ ;

$$\Gamma_v = 0.5 \cdot 10^{-3} \frac{\text{m}^2}{\text{s}}; \quad x_{bw} = 0.1 \text{ m};$$

1 –  $\tau_i = 0, \xi = 270$ ;

2 –  $\tau_i = 3 \frac{\text{N}}{\text{m}^2}, \xi = 400$ ;

3 –  $\tau_i = 10, \xi = 610$ .

A derivative on the wall ( $\eta = 0$ ) of (26) gives

The wall heat flux ( $\eta=0$ ) within the section  $\xi \geq \xi_m$  is during the second period of heating

$$q_2(\xi)_{\eta=0} = -\lambda \frac{t_w - t_{sat}}{\delta} \frac{d\theta}{d\eta_{\eta=0}} = \lambda \frac{t_w - t_{sat}}{\delta} \left\{ \frac{Pe}{8} \left[ \frac{R_1 - 1}{R} + \frac{Pe}{80DR} \frac{16\rho g\delta + 25\tau_i}{3\tau_i + 2\rho g\delta} \exp\left(-\frac{\xi_m}{D}\right) \right] \exp\left(\frac{\xi_m - \xi}{R}\right) + 1 \right\}, \quad (28)$$

and during the first period  $\xi \leq \xi_m$  from (6) will come to

$$q_1(\xi)_{\eta=0} = -\lambda \frac{t_w - t_{sat}}{\delta} \frac{d\theta}{d\eta_{\eta=0}} = \lambda \frac{t_w - t_{sat}}{2\delta\sqrt{\pi}} \sqrt{\frac{Pe}{\xi}}. \quad (29)$$

If the process of liquid film hating up starts right after the passage of a big wave at  $\xi = \xi_o$  and finishes right before the incoming successive big

wave at  $\xi = \xi_v$ , the mean heat fluxes at sections  $(\xi_m - \xi_o)$  and  $(\xi_v - \xi_m)$  will be

$$q_{1av} = \frac{1}{\xi_m - \xi_o} \int_{\xi_o}^{\xi_m} q_1(\xi)_{\eta=0} d\xi = \lambda \frac{t_w - t_{sat}}{\delta\sqrt{\pi}(\xi_m - \xi_o)} (\sqrt{\xi_m Pe} - \sqrt{\xi_o Pe}), \quad (30)$$

$$q_{2av} = \frac{1}{\xi_v - \xi_m} \int_{\xi_m}^{\xi_v} q_2(\xi)_{\eta=0} d\xi = \frac{\lambda(t_w - t_{sat})}{\delta(\xi_v - \xi_m)} \times \left\{ \frac{Pe}{8} (R_1 - 1) \left[ 1 - \exp\left(\frac{\xi_m - \xi_v}{R}\right) \right] + \frac{Pe^2}{640D} \exp\left(-\frac{\xi_m}{D}\right) \left[ 1 - \exp\left(\frac{\xi_m - \xi_v}{R}\right) \right] \left[ \frac{25\tau_i + 16\rho g\delta}{3\tau_i + 2\rho g\delta} \right] + \xi_v - \xi_m \right\}. \quad (31)$$

Now the mean heat flux at the both sections may be given as mean weighted

$$q_{av} = \frac{q_{1av}(\xi_m - \xi_o) + q_{2av}(\xi_v - \xi_m)}{\xi_v - \xi_o}, \quad (32)$$

Then combining (30, 31) we get

$$q_{av} = \lambda \frac{t_w - t_{sat}}{\delta(\xi_v - \xi_o)} \left\{ \frac{1}{\sqrt{\pi}} (\sqrt{\xi_m Pe} - \sqrt{\xi_o Pe}) + \frac{Pe}{8} (R_1 - 1) \left[ 1 - \exp\left(\frac{\xi_m - \xi_v}{R}\right) \right] + \frac{Pe^2}{640D} \exp\left(-\frac{\xi_m}{D}\right) \times \left[ 1 - \exp\left(\frac{\xi_m - \xi_v}{R}\right) \right] \left[ \frac{25\tau_i + 16\rho g\delta}{3\tau_i + 2\rho g\delta} \right] + \xi_v - \xi_m \right\}. \quad (33)$$

If heat transfer coefficient is defined as  $\alpha = \frac{q_{av}}{t_w - t_i}$ , then

$$\alpha = \frac{\lambda}{\delta(\xi_v - \xi_o)} \left\{ \frac{1}{\sqrt{\pi}} (\sqrt{\xi_m Pe} - \sqrt{\xi_o Pe}) + \frac{Pe}{8} (R_1 - 1) \left[ 1 - \exp\left(\frac{\xi_m - \xi_v}{R}\right) \right] + \frac{Pe^2}{640D} \exp\left(-\frac{\xi_m}{D}\right) \times \left[ 1 - \exp\left(\frac{\xi_m - \xi_v}{R}\right) \right] \left[ \frac{25\tau_i + 16\rho g\delta}{3\tau_i + 2\rho g\delta} \right] + \xi_v - \xi_m \right\}, \quad (34)$$

where  $t_i$  – liquid film saturation temperature on the interface. For water films  $t_i = t_{sat}$ . When evaporating from the free surfaces of solution films  $t_i = t_{sat} + \Delta_{fc}$ , where  $\Delta_{fc}$  - physical-chemical depression.

$$x_{bw} = \bar{u}_{ll} \tau = \frac{\bar{u}_{ll} \lambda_{bw}}{c - \bar{u}_{ll}} = \frac{\bar{u}_{ll} c}{f_{bw}(c - \bar{u}_{ll})} = \frac{c}{f_{bw} \left( \frac{c}{\bar{u}_{ll}} - 1 \right)}, \quad (35)$$

where  $c, \lambda_{bw}, f_{bw}$  – face velocity, length and big waves frequency, respectively,

$\tau = \frac{\lambda_{bw}}{c - \bar{u}_{ll}}$  – time, in which the peaks of big waves cross the film cross section, which, in turn, is moving with the  $\bar{u}_{ll}$ .

The values  $c, \lambda_{bw}, f_{bw}$  are the function of wetting density, interface shear stress and channels geometry. Therefore, the analysis of heat transfer in liquid films covered by big waves can be done with the adequate accuracy with the application of data characterizing wave processes only in channels with the relevant geometry and the experimental conditions that correspond to those, taken for the analysis. The length of big waves  $\lambda_{bw}$  on the water film surface in a tube with 25 mm diameter is according to [8]. The Reynolds number  $R=40\dots400$  remains approximately constant within  $\lambda_{bw} = 100\dots120$  mm, with the increase of the Reynolds number to 4000 the length of big waves also increased gradually to 140 mm at distance 2.38 m from the heat transfer coefficients with the experimental data, obtained when studying heat transfer to the saturated liquid films at the regime of evaporation from free surface.

In [12] the expression for  $\xi_o$  applicable to the free falling water and sugar solution films in a tube of 20 mm

It should be pointed out that  $\xi_v - \xi_o$  corresponds to the dimensionless  $\frac{x_{bw}}{\delta}$ , which is to be passed by the last layer of the film when moving with the velocity  $\bar{u}_{ll}$  between the peaks of big waves

distributing device. At these conditions the phase velocity  $c$  was 0.5...2.5 times the mean film velocity  $\bar{u}_{ll}$ .

From (35) follows that given  $c = 2\bar{u}_{ll}$  the distance, which the film passes within the time of a big wave passage will be equal to the length of a big wave, that is  $x_{bw} = \lambda_{bw}$ . Therefore, it is possible to assume approximately that

$$\xi_v - \xi_o = \frac{\lambda_{bw}}{\delta}.$$

As it may be concluded from (33), the decrease in  $\xi_o$  entails growth of the heat flux, insofar there is a noticeable growth of the temperature gradient on the wall, and vice versa. Therefore, for a given  $\lambda_{bw}$  the value  $\xi_o$  remains the only sought function in (34), which correlates the relationship between the modeled heat transfer characteristics with experimental data. The function  $\xi_o$  has been derived based upon the comparison of the calculated by the correlation (34)

diameter and 1.8 m at  $\lambda_{bw}=120$  mm is given

$$\begin{aligned} \xi_o &= 43Y + 0.2 \quad \text{при } Y \leq 0.115, \\ \xi_o &= 1150Y^{0.98} - 133, \quad \text{при } \\ Y &\geq 0.115, \end{aligned} \quad (36)$$

$$\text{where } Y = \left( \frac{\sqrt[3]{3\Gamma_v \nu}}{\delta^*} \right)^{0.9} \left( \frac{\nu_{\text{wat}}}{\nu} \right)^{0.01}; \nu_{\text{wat}}$$

– water cinematic coefficient of viscosity;  $\delta^* = \sqrt{\frac{\sigma}{g\rho}}$  – capillary constant;  $\sigma$  – surface tension.

At  $Pe \geq Pe_o$ ,

$$Y_o = \left( \sqrt[3]{\frac{3Pe_o a \nu}{4g}} / \delta^* \right)^{0.9} \left( \frac{\nu_{\text{wat}}}{\nu} \right)^{0.01},$$

$$\xi_o = 43Y_o + 0.2, \text{ at } Y \leq 0.115$$

$$\xi_o = 1150Y_o^{0.98} - 133, \text{ at}$$

$$Y \geq 0.115$$

$$\zeta = \zeta_1 + 627 \left( \frac{d_o}{d} \right) \left[ Fr_2^{1.26} \exp \left( 1.25 \cdot 10^{-2} K_\delta^{1.5} \left( Fr_2 - H^{1.1} \sqrt{\frac{d}{d_o}} \right) \right)^{-1} - Fr_2^{1.26} \right]^{-1},$$

(38)

where  $\zeta_1 = \zeta_c + 3 \cdot 10^{-3} + 4 \cdot 10^{-2} K_\delta$  – hydraulic friction coefficient for the first region – region of weak interaction;

$\zeta_c = \frac{0.316}{Re_2^{0.25}}$  – hydraulic friction on dry

wall;  $K_\delta = \sqrt[6]{\frac{\Gamma_v^3 \nu}{g^2}} \sqrt{\frac{g\rho}{\sigma}}$ ;  $H = \sqrt{\frac{\rho\sigma}{gd^2\rho_2^2}}$

;  $Fr_2 = \frac{u_2^2}{gd}$ ;  $Re_2 = \frac{u_2 d \rho_2}{\mu_2}$ ;  $d_o = 0.013 \text{ m}$

;  $\mu_2$  – steam dynamic viscosity;  $d$  – tube diameter.

Transfer between the first and second regions happens at a condition

In case of the co-current steam flow a shear stress appears on the interface, which is determined as

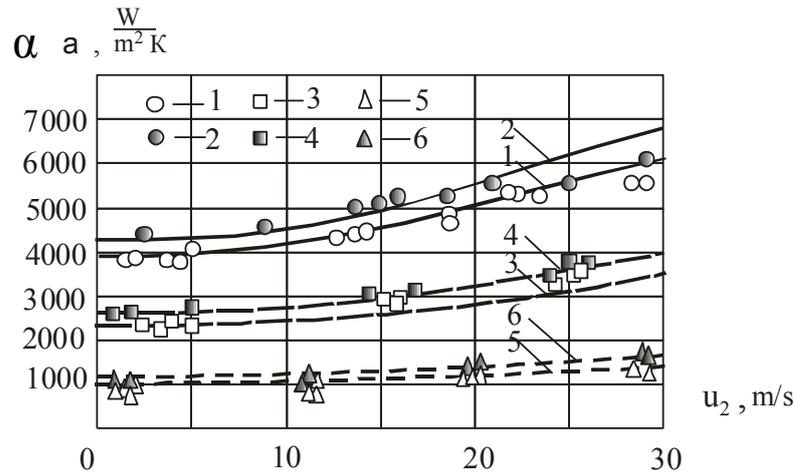
$$\tau_i = \zeta \rho_2 \frac{u_2^2}{8}, \quad (37)$$

Where  $u_2$  – steam core velocity;  $\rho_2$  – steam density.

The value of interface friction coefficient can be determined from [11]. The correlation generalizes the experiments on the determination of friction pressure loss in steam- and gas-liquid flows of water and sugar solutions in tubes with diameter 13...55mm

$$Fr_2 - H^{1.1} \sqrt{\frac{d}{d_o}} \geq 0.$$

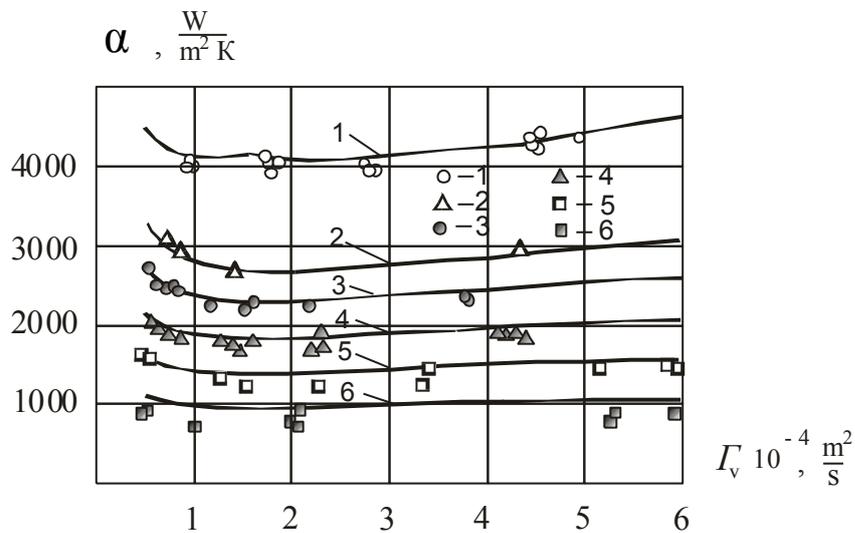
A comparison of experimental data on heat transfer coefficients to water and sugar solutions with the concentration up to 70% DM ( $\nu = 3.77 \cdot 10^{-6} \text{ m}^2/\text{s}$ ) has been made. The data obtained from the regime of falling dawn films over the vertical surface with the interface interaction from [11] with the results calculated by (3, 34, 36, 37, 38) are given in Fig. 5.



**Figure 5. Effect of steam core velocity upon heat transfer coefficient. Water and sugar solutions at  $t = 100\text{ }^{\circ}\text{C}$ . 1, 2 – water; 3, 4 – sugar solution, DM=40%; 5, 6 – sugar solution, DM = 70%.  $\lambda_{bw}=120\text{ mm}$ . Lines – calculated by (3, 34, 36, 37, 38)**

The derived equations (34, 36) reflect the heat transfer mechanism in films, in case there is no co-current steam core ( $u_2=0, \tau_i=0$ ).

Fig. 6 illustrates the comparison of calculations by (34, 36) with the experimental heat transfer coefficients obtained from free falling water and sugar solutions (up to 70% DM) on the vertical surface [11].



**Figure 6. Comparison of experimental heat transfer coefficients (HTC) to water and sugar solutions free falling films with the calculated results by (34, 36) at  $\lambda_{bw}=120\text{ mm}$ . 1 – water,  $t = 100\text{ }^{\circ}\text{C}$ ; 2 – sugar solution, DM = 30 %; 3 – 40 %; 4 – 50 %; 5 – 60 %; 6 – 70 %.**

As it can be seen from the given plots, the correlations (34, 36) are also valid within the turbulent range

( $R=6500$  for water) despite the fact that the turbulence parameters were not used explicitly in the derived equations.

#### 4. Conclusions

1. A model of heat transfer to the liquid films with the developed wavy structure down flowing on the vertical surfaces has been developed. The proposed model is based on the suggestions that the periodic relaxation of temperature field, disturbed as a result of a mixing effect of big waves. The comparison of experimental results with the calculated within the proposed model ones seem to be acceptable. This coincidence proves the applicability of the proposed model and describes adequately the

mechanism of heat transfer to the films heated to the saturation temperature with the developed wavy structure.

2. The equations (3, 34, 36, 37, 38) are recommended to be used for the calculation of HTC in water and sugar solutions (up to 70% DM) at a regime of free falling and the presence of co-current steam flow, as well. The equations may be applied to the films at a regime of evaporation from the interface of laminar and turbulent films within the marked above range of flow rates.

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